Chemistry

Time:2hours **Marks:60**

1. Five isomeric para-disubtituted aromatic compounds **A** to **E** with molecular formula $C_8H_8O_2$ were given for identification. Based on the following observations, give structures of the compounds.
   (I) Both **A** and **B** form a silver mirror with Tollen's reagent; also **B** gives a positive test with $FeCl_3$ Solution.
   (II) **C** gives positive Idoform test.
   (III) **D** is readily extracted n aqueous $NaHCO_3$ solution.
   (IV) **E** on acid hydrolysis gives 1,4-dihydroxybenzene. [5]

2. 500 mL of 0.2 M aqueous solution of acetic acid is mixed with 500 mL of 0.2 M HCl at 25°C.
   (I) Calculate the degree of dissociation of acetic acid in the resulting solution and pH of the solution.
   (II) If 6 g of NaOH is added to the above solution,determine the final pH. [Assume there is no change in volume on mixing; Ka of acetic acid is 1.75×10^{-5} mol L^{-1}]. [5]

3. Deduce the structures of $[NiCl_4]^{2-}$ and $[Ni(CN)_4]^{2-}$ considering the hybridization of the metal ion. Calculate the magnetic moment (spin only) of the species. [5]

4. The density of the vapour of a substance at 1 atm pressure and 500 K is 0.36 kg m^{-3}. The vapour effuses through a small hole at a rate of 1.33 times faster than oxygen under the same condition.
   (a) Determine (I) molecular weight, (II) molar volume, (III) compression factor (Z) of the vapour and (IV) which forces among the gas molecules are dominating, the attractive or the repulsive?
   (b) If the vapour behaves ideally at 1000K, determine the average translational kinetic energy of a molecule. [5]

5. Write balance equations for the reactionsof the following compounds with water;
   (I) Al_4C_3 (II) CaNCN (III) BF_3
   (IV) NCl_3 (V) XeF_4 [5]

6. Identify X, Y and Z in the following synthetic scheme and write their structures.

$$CH_3CH_2C\equiv C-H \xrightarrow[(II)CH_3CH_2Br]{(I)NaNH_2} X \xrightarrow{H_2/Pd\text{-}BaSO_4} Y \xrightarrow{\text{alkaline}KMnO_4} Z$$

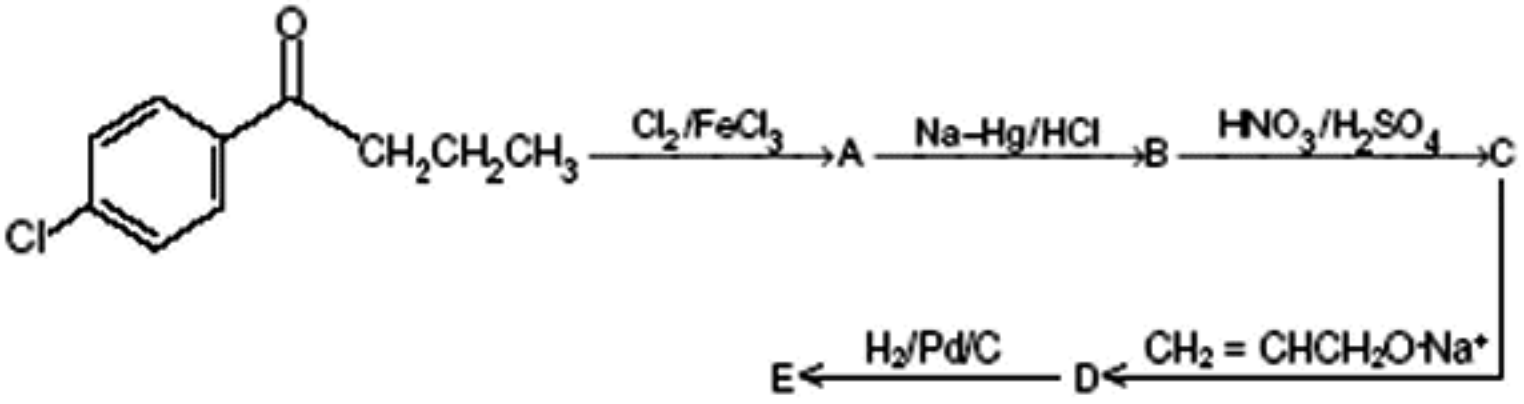
   Is the compound Z optically active? Justify your answer. [5]

7. How is boron obtianed from borax? Give chemical equations with reaction conditions. Write the structure of B_2H_6 and its reaction with HCl.

8. ^{64}Cu (half-life = 12.8h) decays by β^- emission (38%), β^+ emission (19%) and electron capture (43%). Write the decay products and calculate partial half-lives for each of the decay processes. [5]

9. When a white crystaline compound **X** is heated with $K_2Cr_2O_7$ and concentrated H_2SO_4, a reddish brown gas **A** is evolved. On passing **A** into caustic soda solution, a yellow coloured solution of **B** is obtained. Neutralizing the solution **B** with acetic acid and on subsequent addition of lead acetate, a yellow precipitate **C** is obtained. When **X** is heated with NaOH solution, a colourless gas is evolved and on pasing this gas into K_2HgI_4 solution, a reddish brown precipitate **D** is formed, identify **A.B,C,D** and **X**. Write the equations of reactions involved. [5]

10. Write structures of the product **A,B,C,D** and **E** in the following scheme.



[5]

11. A biologically active compound, Bombykol ($C_{16}H_{30}O$) is obtained from a natural source. The structure of the compound is determined bythe following reactions.
(a) On hydrogenation, Bombykol gives a compound **A**, $C_{16}H_{34}O$, which reacts with acetic anhydride to give an ester;
(b) Bombykol also reacts with acetic anhydride to give another ester, which on oxidative ozonolysis (O_3/H_2O_2) gives a mixture of butanoic acid, oxalic acid and 10-acetoxy decanioc acid.
Determine the number of double bonds in Bombykol. Write the structures of compound **A** and Bombykol. How many geometrical isomers ar possible for Bombykol? [5]

12. Two moles of a perfect gas undergo the following processes:
(a) a reversible isobaric expansion from (1.0 atm, 20.0L) to (1.0 atm, 40.0L);
(b) a reversible isochoric change of state from (1.0 atm, 40.0 L) to (0.5 atm, 40.0 L);
(c) a reversible isothermal compression from (0.5 atm, 40.0 L) to (1.0 atm, 20.0 L).
(I) Sketch with lables each of the processes on the same P-V diagram.
(II) Calculate the total work (w) and the total heat change (q) involved in the above processes.
(III) What will be the valuesof ΔU, ΔH and ΔS for teh overall process? [5]

57. The sides of a triangle are in the ratio $1:\sqrt{3}:2$, then the angles of the triangle are in the ratio

(a) 1 : 3 : 5 (b) 2 : 3 : 4 (c) 3 : 2 : 1 (d) 1 : 2 : 3

58. Area of triangle formed by the lines $x + y = 3$ and angle bisectors of the pair of straight lines $x^2 - y^2 + 2y = 1$ is

(a) 2 sq. units (b) 4 sq. units (c) 6 sq. units (d) 8 sq. units

59. If three distinct numbers are chosen randomly from the first 100 natural numbers, then the probability that all three of them are divisible by both 2 and 3 is

(a) $\frac{4}{55}$ (b) $\frac{4}{35}$ (c) $\frac{4}{33}$ (d) $\frac{4}{1155}$

60. The area enclosed between the curves $y = ax^2$ and $x = ay^2$ $(a > 0)$ is 1 sq. unit. Then the value of 'a' is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{3}$

61. Given both θ and ϕ are acute angles $\sin\theta=\frac{1}{2}, \cos\phi=\frac{1}{3}$, then the value of $\theta+\phi$ belongs to

(a) $\left(\frac{\pi}{3},\frac{\pi}{2}\right]$ (b) $\left(\frac{\pi}{2},\frac{2\pi}{3}\right)$ (c) $\left(\frac{2\pi}{3},\frac{5\pi}{6}\right]$ (d) $\left(\frac{5\pi}{6},\pi\right]$

62. If tangents are drawn to the ellipse $x^2 + 2y^2 = 2$, then the locus of the mid-point of the intercept made by the tangents between the coordinate axes is

(a) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (b) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$ (c) $\frac{x^2}{2}+\frac{y^2}{4}=1$ (d) $\frac{x^2}{4}+\frac{y^2}{2}=1$

63. If f(x) is differentiable and $\int_0^{t^2} xf(x)dx=\frac{2}{5}t^5$ then $f\left(\frac{4}{25}\right)$ equals

(a) $\frac{2}{5}$ (b) $\frac{-5}{2}$ (c) 1 (d) $\frac{5}{2}$

64. The value of x for which $\sin(\cot^{-1}(1 + x)) = \cos(\tan^{-1} x)$ is

(A) $\frac{1}{2}$ (b) 1 (c) 0 (d) $-\frac{1}{2}$

65. If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$

(a) f(x) is strictly increasing function (b) f(x) has a local maxima
(c) f(x) is strictly decreasing function (d) f(x) is bounded

66. If $\omega(\neq 1)$ be a cube root of unity and $(1+\omega^2)^n = (1+\omega^4)^n$, then the least positive value of n is

(a) 2 (b) 3 (c) 5 (d) 6

67. If $f(x) = x^{\alpha} \log x$ and $f(0) = 0$, then the value of α for which Rolle's theorem can be applied in [0, 1] is

(a) -2 (b) -1 (c) 0 (d) ½

10. Let V be the volume of the parallelopiped formed by the vectors $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$,. If ar, br, cr, where r = 1, 2, 3, are nonnegative real numbers and $\sum_{r=1}^{3}(a_r+b_r+c_r)=3L$, show that $V\le L^3$.

11. For any natural number m, evaluate $\int(x^{3n}+x^{2n}+x^n)(2x^{2n}+3x^m+6)^{1/n}dx, x>0$.

12. Let $f(x)=\begin{cases}x+a & \text{if } x<0\\ |x-1| & \text{if } x\ge 0,\end{cases}$ and $g(x)=\begin{cases}x+1 & \text{if } x<0\\ (x-1)^2+b & \text{if } x\ge 0,\end{cases}$

where a and b are nonegative real numbers. Determine the composite function g of. If (gof) (x) is continuous for all real x, determine the values of a and b. Further, for these values of a and b, is g of differentiable at x = 0 ? Justify your answer.

Physics

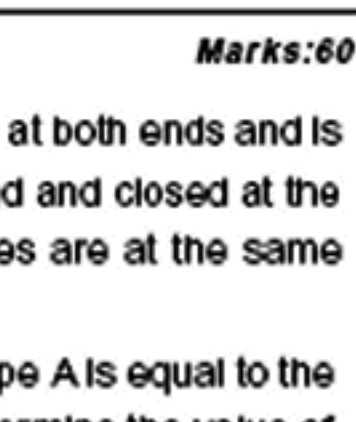
Time:2hours **Marks:60**

1. Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monoatomic gas of molar mass M_A. Pipe B is open at opne end and closed at the other end, and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature.
   (a) If the frequency of the second harmonic of the fundamental mode in Pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B, determine the value of M_A/M_B.
   (b) Now the opeen end fo pipe B is also closed (so that the pipe is closed at both ends). Find the ratio of the fundamental frequency in pipe A to that in pipe B. [3+2]

2. A cubical box of side 1 meter contains helium gas (atomic weight 4) at a pressure of 100N/m^2. During an observation time of 1 second, an atom travelling with the root-mean-square speed parallel to oneof the edges of the cube, was found to make 500 hits with a particular wall, wolthout any collision with other atoms.

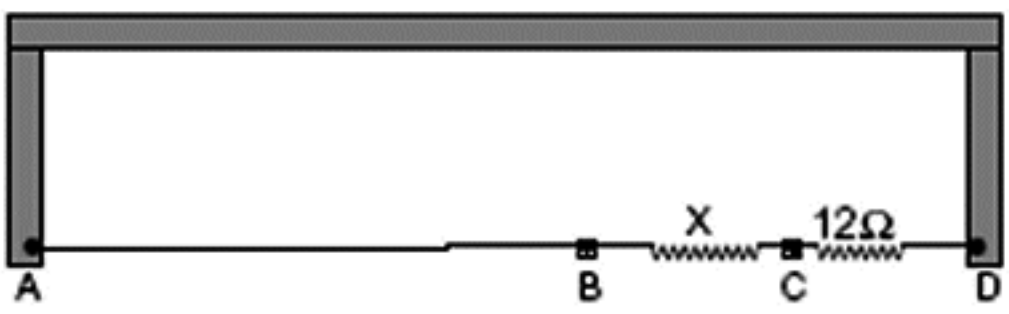
   Take $R = \frac{25}{3}$ J/mol−K and $k = 1.38\times10^{-23}$ J/K
   (a) Evaluate the temperature of the gas.
   (b) Evaluate the average kinetic energy per atom.
   (c) Evaluate the total mass of helium gas in the box. [2+1+2]

3. A uniform solid cylinder of density 0.8 g/cm^3 floats in equilibrium in a combination of two non - mixing liquids A and B with its axis vertical. The densities of the liquids A and B are 0.7 g/cm^3 and 1.2 g/cm^3, respectively. The height of liquid A is h_A = 1.2 cm. The length of the part of the cylinder immersed in liquid B is h_B = 0.6 cm.



   (a) Find the total force exerted by liquid A on the cylinder.
   (b) Find h, the length of the part of teh cylinder in air.
   (c) The cylinder is depressed in such a way that is top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder immediately after it is released. [1+2+2]

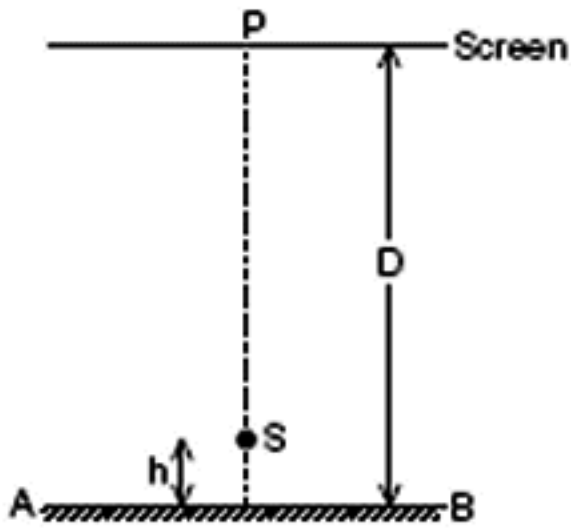
4. A thin uniform wire AB of length 1m, an unknown resistance X and a resistance of 12Ω are connected by thick conducting strips, as shown in the figure. A battery and a galvanometer (with a sliding jockey connected to it) are also available Connections are to be made to measure the unknown resistance X using the principle of Wheatstone bridge. Answer the following questions.



(a) Are there positive and negative terminals on the galvanometer?
(b) Copy the figure in your answer book and show the battery and the galvanometer (with jockey) connected at appropriate points.
(c) After appropriate connections are made, it is found that no deflection takes place in the galvanometer when the sliding jockey touches the wire at a distacne of 60 cm from A. Obtain the valueof the resistance of [2+1+2]

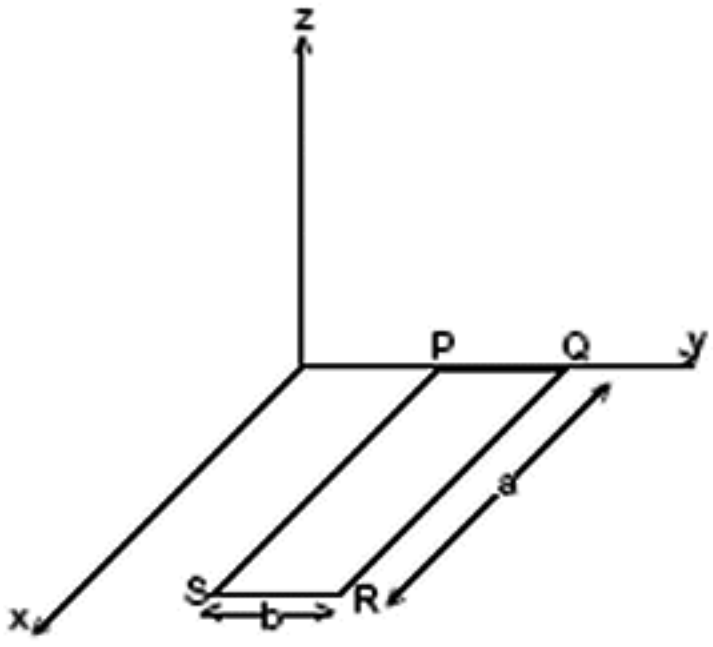
5. A hydrogen-like atom (described by the Bohr model) is observed to emit six wavelengths, originating from all possible transitions between a group of levels. These levels have energies between -0.085 eV and -0.544 eV (**including** both these values).
(a) Find the atomic number of the atom.
(b) Calculate the smallest wavelength emitted in these transitions.
(Take hc = 1240 eV-nm, ground state energy of hydrogen atom = -13.6 eV) [4+1]

6. A point source S emitting light of wavelength 600 nm is placed at a very small height h above a flat reflecting surface AB (See figure). The intensityof the reflected light is 36% of the incident intensity. Interference fringes are observed on a screen placed parallel to the reflecting surface at a very large distacne D from it.



(a) What is the shape of the interference fringes on the screen?
(b) Calculate the ratio of the minimum to the maximum intensities in the interference fringes formed near the point P (shown in the figure).
(c) If the intensity at point P corresponds to a maximu, calculate the minimum distance through which the reflecting surface AB should be shifted so that the intensity at P agian becomes maximum. [1+3+1]

7. A rectangular loop PQRS made from a uniform wire has length a, width b and mass m. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the y-axis (see figure). Take the vertically upward direction as the z-axis. A uniform magnetic field $\vec{B} = (3\hat{i} + 4\hat{k})B_0$ exists in the region. The loop is held in the x-y plane and a current I is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium.



(a) What is the direction of the current I in PQ?
(b) Find the magnetic force on the arm RS.
(c) Find the expression for *I* in terms of B_0, a, b and m. [1+1+3]

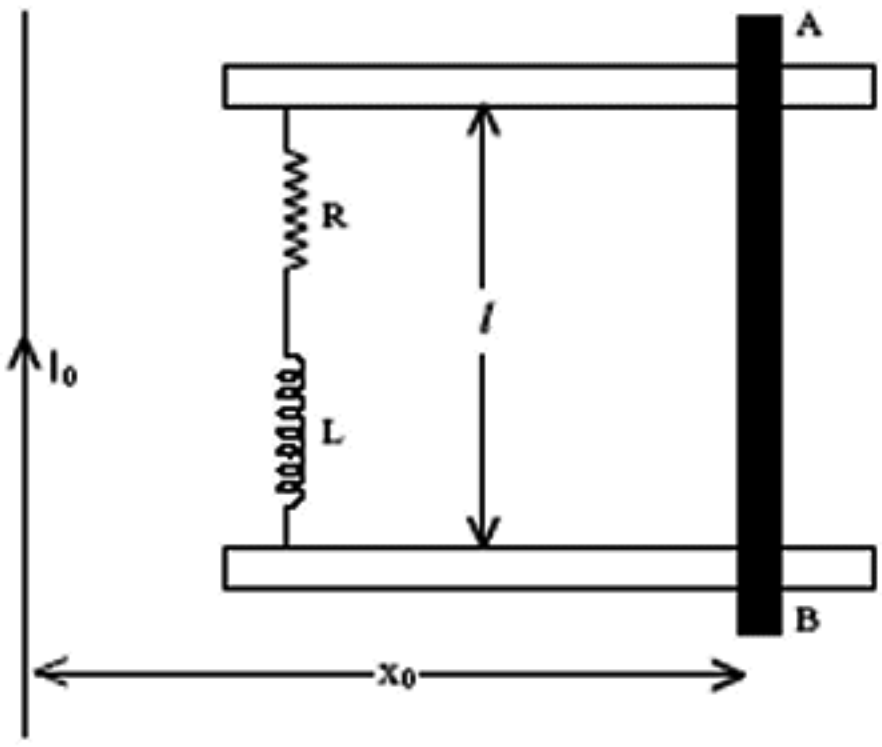
8. Two metallic plates A and B, each of area $5\times10^{-4}\,m^2$, are placed parallel to each other ata separation of 1 cm. Plate B carries a positive charge of 33.7×10^{-12}C. A monochroomatic beam of light, with photons of energy 5 eV ecah, starts falling on plate A at t = 0 so that 10^{16} photons fall on it per square meter per second. Assume that one photoelectron is emitted for every 10^6 incident photons. Also asume that all the emitted photoelectrons are collected by plate B and the work function of plate A remains constant at the value 2eV. Determine

(a) the numberof photoelectrons emitted up to t = 10 s,
(b) the magnitude of the elecric field between the plates A and B at t = 10 s, and
(c) the kinetic energy of the most energetic photoelectron emitted at t = 10 s when it reaches plate B.

Neglect the time taken by the photoelectron to reach plate B.

Take $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N-m^2$ [1+2+2]

9. A metal bar AB can slide on two parallel thick metallic rails separated by a distance *l*. A resistance R and an inductance *L* are connected to the rails as shwon in the figure. A long straight wire carrying a constant current I_0 is placed in the plane of the rails and perpendicular to them as shwon. The bar AB is held at rest at a distance x_0 from the long wire. At t =0, it is made to slide on the rails away from the wire. Answer the following questions.

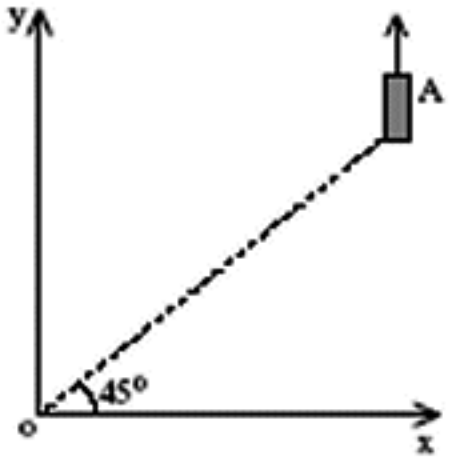


(a) Find a relation among i, $\frac{di}{dt}$ and $\frac{d\phi}{dt}$, where I is the current in the circuit and ϕ is the flux of the magnetic field due to the long wire through the circuit.

(b) It is observed that at time t = T, the metal bar AB is at a distnace of $2x_0$ from the long wire and the resistance R carries a current I_1. Obtain an expression for the net charge that has flown through resistance R from t = 0 to t = T.

(c) The bar is suddenly stoped at time T. The current through resistance R is found to be $\frac{i_1}{4}$ at time 2T. Find the value of L/R in terms of the other given quantities. D[1+2+2]

10. On frictionless horizontal surface, assumed to be the x-y plane, a small trolley A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $(\sqrt{3}-1)$ m/s. At a particular instant, when the line OA makes an angle of 45^0 with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley.



(a) The motion of the ball is observed from the frame of the trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.

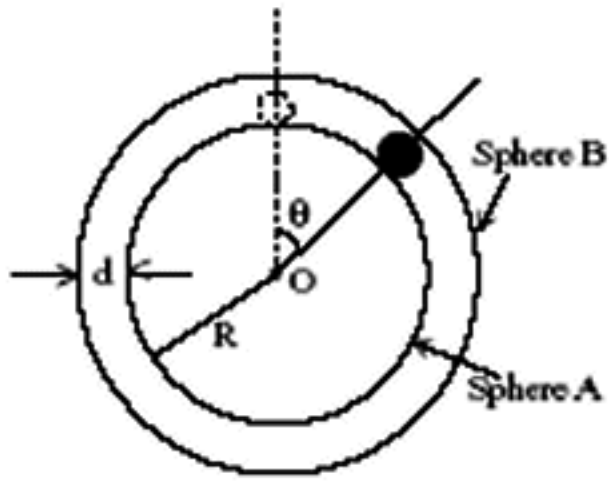
(b) Find the speed of the ball with respect to the surface, if $\phi = 4\theta/3$ [2+3]

11. A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B (See figure). The smaller sphere A has a radius R and the space between the two spheres has a width d. The ball has a diameter **very slightly** less than d. All surfaces are frictionless. The ball is given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is denoted by θ (shown in figure).

(a) Express the total normal reactionforce exerted by the sphere on the bal as a function of

angle θ.

(b) Let NA and NB denote the magnitudes of the normal reaction forces on the ball exerted by the sphere A and B, respectively. Sketch the variations of NA and NB as functions of $\cos\theta$ in the range $0 \le \theta \le \pi$ by drawing two separate graphs in your answre book, taking $\cos\theta$ on the hortzontal axes. [2+3]



12. Three particles A, B and C, each of mass m, are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side *l*. This body is placed on a horizontal frictionless table (x-y plane) and is hinged to it at the point A so that it can move without friction about the vertical axis through A (See figure). The body is set into rotational motion on the table about A with a constant angular velocity ω.

(a) Find the magnitude of the horizontal force exerted by the hinge on the body.

(b) At time T, when the side BC is parallel to the x-axis, a force F is applied on B along BC (as shown). Obtain the x-component and the y-component of the force exerted by the hinge on the body, immediately after time T. [1+4]