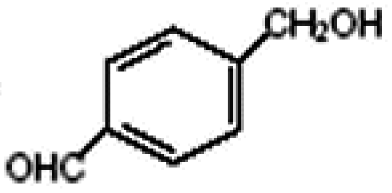
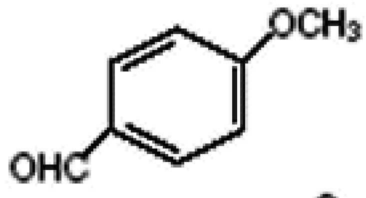
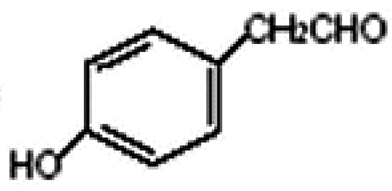
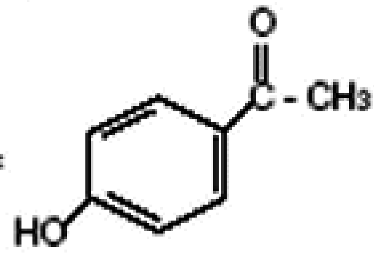
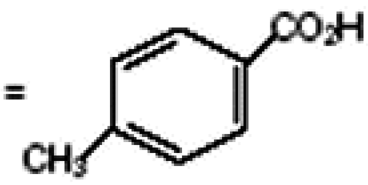
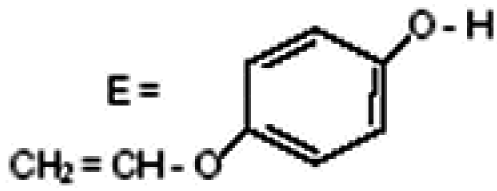
1. A =  or 

B =  C = 

D =  E = 

2. (i) The volume being doubled by mixing the two solutions, the molarity of each component will be halved i.e.

$[CH_3COOH] = 0.1$ M, $[HCl] = 0.1$ M

HCl being a strong acid will remain completely ionised and hence H^+ ion concentration furnished by it will be 0.1 M. This would exert common ion effect on the dissociation of acetic acid, a weak acid.

CH_3COOH	$\rightleftharpoons$	CH_3COO^- +	H^+
C		0	0
$C(1-\alpha)$		$C\alpha$	$C\alpha + 0.1$

$$K_a = \frac{C\alpha\alpha(C+0.1)}{C(1-\alpha)} = \frac{C\alpha^2 + 0.1\alpha}{(1-\alpha)}$$

Neglecting α in comparison to unity and $C\alpha^2$ i.e. $0.1\alpha^2$ in comparison to 0.1α, we get $K_a = 0.1\alpha$

$$\text{or } \alpha = \frac{K_a}{0.1} = \frac{1.75\times10^{-5}}{0.1} = 1.75\times10^{-4}$$

$[H^+]_{Total} = 0.1 + C\alpha$, $C\alpha$ is negligible as compared to 0.1

$\therefore [H^+]_{Total} = 0.1$

$\therefore pH = 1$

(II) 6 g NaOH = $\frac{6}{40}$ = 0.15mol

0.1 mole of NaOH will be consumed by 0.1 mole of HCl. Thus, 0.05 mole of NaOH will react with acetic acid (No. of mol of $CH_3COOH = 1 \times 0.1 = 0.1$) according to the equation.

CH_3COOH	+	NaOH	$\longrightarrow$	$CH_3COOCNa$	+	H_2O
0.1 mol		0.05 mol		0		0
0.05 mol		0 mol		0.05 mol		0.05 mol

Thus, solution of acetic acid and sodium acetate will become acidic buffer. So pH of the buffer wil be

$$pH = pK_a + \log\frac{[salt]}{[acid]} = -\log(1.75\times10^{-5}) + \log 1 = 4.75$$

3. $[NiCl_4]^{2-}$ $\Rightarrow$ sp^3 (as Cl^- is weak field ligand) - Tetrahedral

$[Ni(CN_4]^{2-}$ $\Rightarrow$ dsp^2 (as CN^- is strong field ligand) - Square planar

Magnetic moments (μ_{spin}) values are as follows,

$[NiCl_4]^{2-}$ $\Rightarrow \sqrt{2(2+2)} = 2.82$ B.M

4. a) $d = 0.36$ kg m^{-3} = 0 .36g/L

(I) From Graham's Law of diffusion

$$\frac{r_v}{r_{O_2}} = \sqrt{\frac{M_{O_2}}{M_v}}; 1.33 = \sqrt{\frac{32}{M_v}} \therefore M_v = \frac{32}{(1.33)^2} = 18.09; \text{ where } M_v = \text{MW of the vapour}$$

(II) Thus, 0.36g = $\frac{0.36}{18.09}$mol

$\frac{0.36}{18.09}$mol mol occupies 1 L volume

so 1 mol occupies $\frac{18.09}{0.36}$L

Thus, molar volume of vapour = 50.25 L

Assuming ideal behaviour the volume of the vapour would be

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow V_2 = 22.4\times\frac{500}{273} = 41.025L$$

(II) Compressibility factor $(Z) = \frac{(PV)_{obs}}{(PV)_{ideal}} = \frac{1\times50.25}{1\times41.025} = 1.224$

(IV) Z being greater than unity, it is the short range repulsive force that would dominate.

(b) $E = \frac{3}{2}k_B T = \frac{3}{2}\times1.38\times10^{-23}\times1000 = 2.07\times10^{-20}$ J per molecule.

5. (I) $Al_4C_3 + 12H_2O \longrightarrow 4Al(OH_3) + 3CH_4 \uparrow$

(II) $CaNCN + 3H_2O \longrightarrow CaCO_3 \downarrow + 2NH_3$

Ammonia formed dissolves in water to form NH_4OH

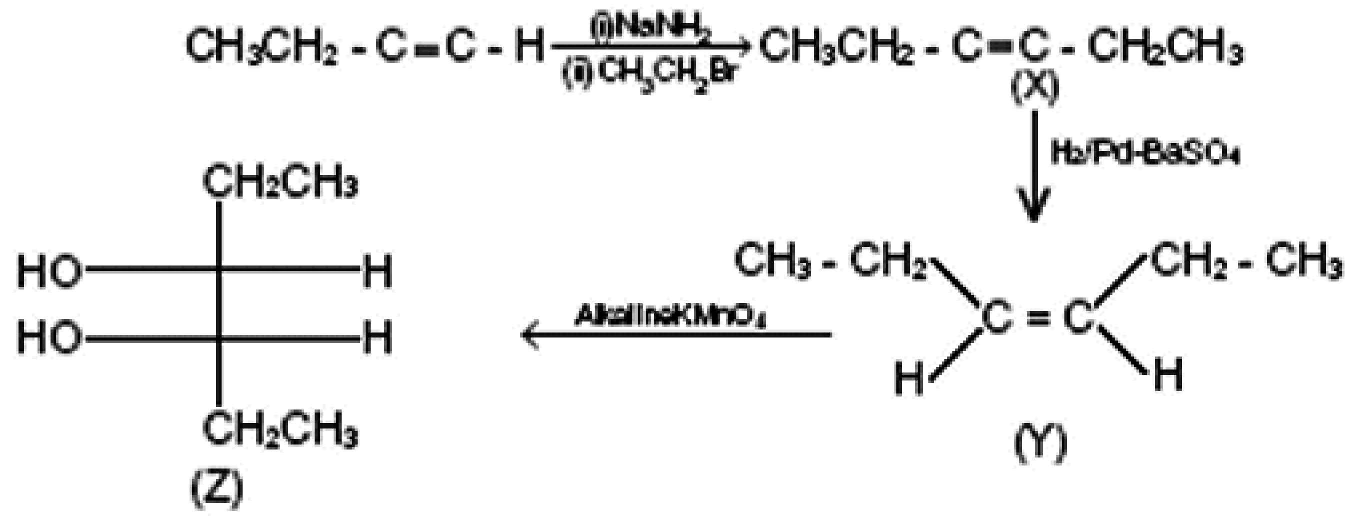
$CaNCN + 5H_2O \longrightarrow 2NH_4OH + CaCO_3 \downarrow$

(III) $4BF_3 + 3H_2O \longrightarrow 3HBF_4 + B(OH)_3$

(IV) $NCl_3 + 3H_2O \longrightarrow NH_3 + 3HOCl$

(V) $3XeF_4 + 6H_2O \longrightarrow XeO_3 + 2Xe + 3/2O_2 + 12HF$

6.



Z is in meso form having plane of symmetry. The upper half molecule is mirror image to the lower half molecule. The molecule is, therefore, optically inactive due to internal compensation.

7. When hot concentrated HCl is added to borax ($Na_2B_4O_7.10H_2O$) the sparingly soluble H_3BO_3 is formed which on subsequent heating gives B_2O_3 which is reduced to boron on heating with Mg, Na or K.

$Na_2B_4O_7$(anhydrous) $+ 2HCl \longrightarrow 2NaCl + H_2B_4O_7$

$H_2B_4O_7 + 5H_2O \longrightarrow 4H_3BO_3$

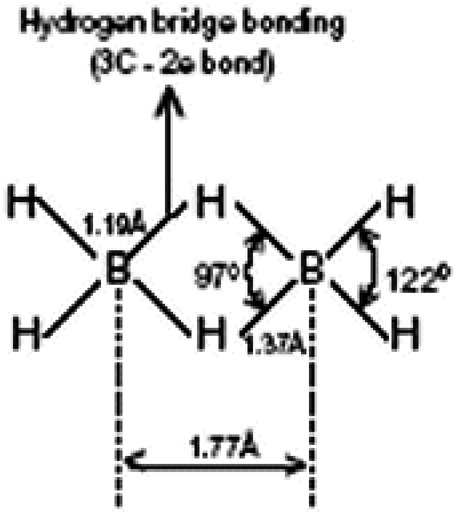
$2H_3BO_3 \xrightarrow{\text{strong heating}} B_2O_3 + 3H_2O$

$B_2O_3 + 6K \longrightarrow 2B + 3K_2O$

or $B_2O_3 + 6Na \longrightarrow 2B + 3Na_2O$

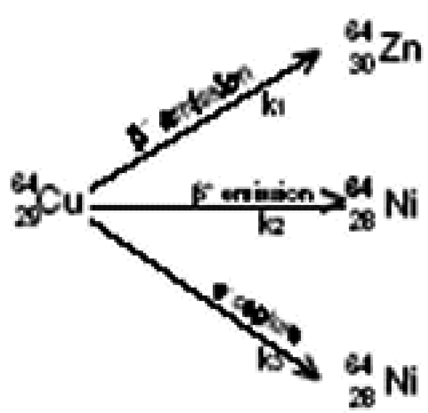
or $B_2O_3 + 3Mg \longrightarrow 2B + 3MgO$

Structure of B_2H_6



$B_2H_6 + HCl \longrightarrow B_2H_5Cl + H_2$,

Normally this reaction takes place in the presence of Lewis acid ($AlCl_3$)

8. 

Let the rate constants of the above emission processes be k_1, k_2 and k_3, respectively and the overall rate constant be k. Then

$$k = k_1 + k_2 + k_3 = \frac{0.693}{t_{1/2}} = \frac{0.693}{12.8} h^{-1}$$

Also, $k_1 = 0.38\ k = 0.38 \times \frac{0.693}{12.8} h^{-1}$

$$t_1 = \frac{0.693 \times 12.8}{0.38 \times 0.693} = 33.68 h$$

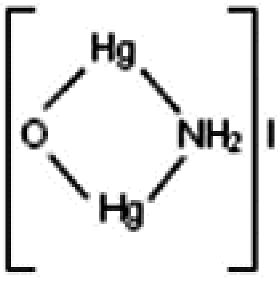
Similarly,

$$t_2 = \frac{0.693}{k_2} = \frac{0.693}{0.19k} = \frac{0.693}{0.19 \times 0.693} \times 12.8 = 67.36 h$$

$$t_3 = \frac{0.693}{k_3} = \frac{0.693}{0.43k} = \frac{12.8}{0.43} = 29.76 h$$

where t_1, t_2 and t_3 are the partial half lives for β^- emission, β^+ emission and electron capture processes, respectively.

9. X = NH_4Cl A = CrO_2Cl_2

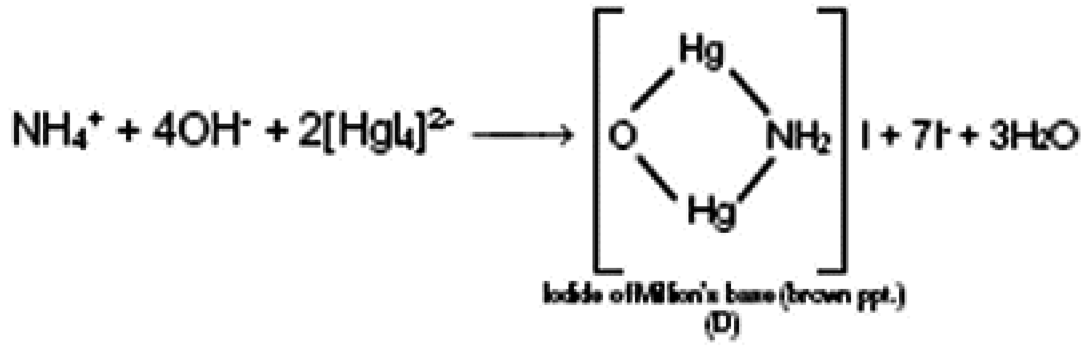
B = Na_2CrO_4 C = $PbCrO_4$ D = 

The chemical reactions are as follows:

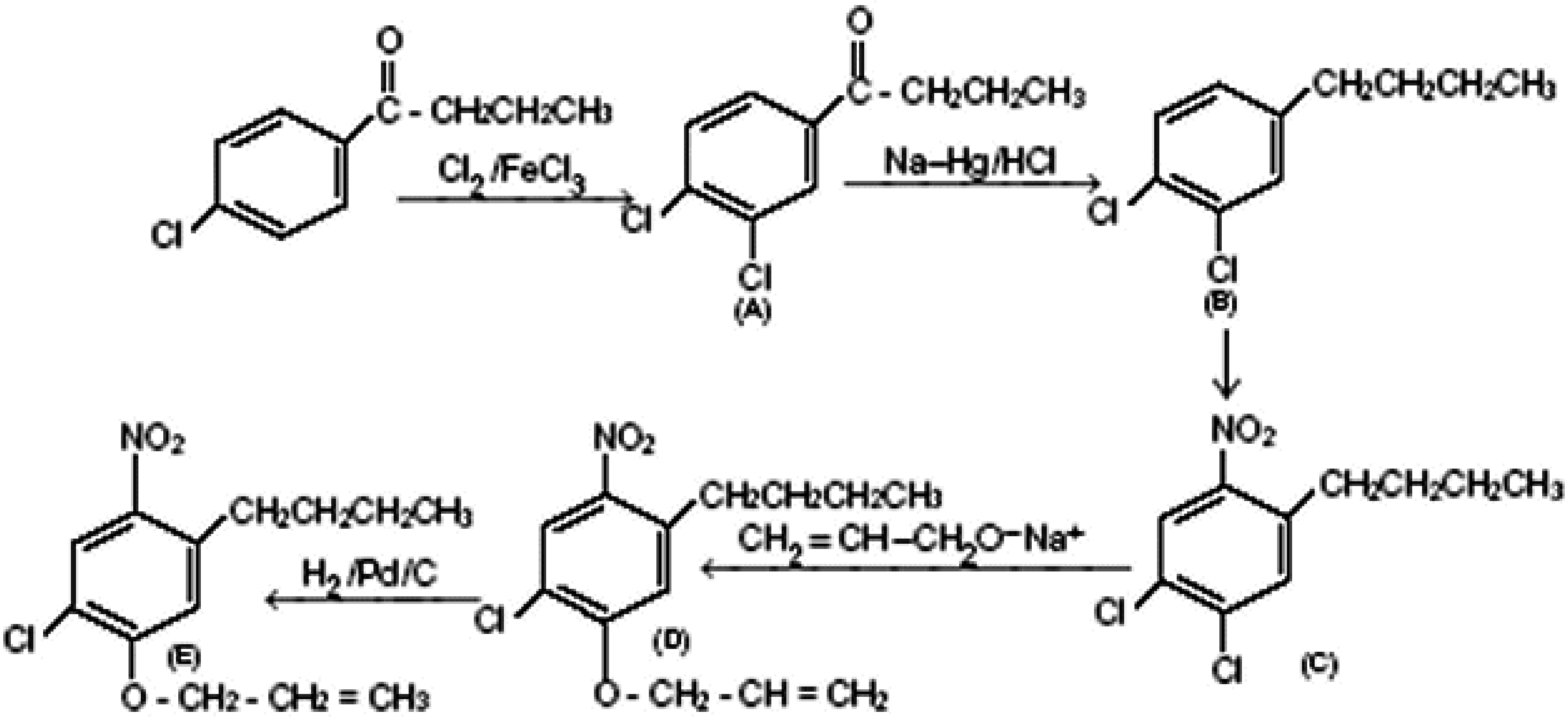
$$Cr_2O_7^{-2} + 4Cl^- + 6H^+ \longrightarrow \underset{\text{(Reddish Brown)}\ (A)}{2CrO_2Cl_2}\uparrow + 3H_2O$$

$$CrO_2Cl_2 + 4OH^- \longrightarrow \underset{\substack{\text{(Yellow Solution)} \\ (B)}}{CrO_4^{-2}} + 2Cl^- + 2H_2O$$

$$Pb(OCOCH_3)_2 + CrO_4^{-2} \longrightarrow \underset{\substack{\text{(Yellow ppt)} \\ (C)}}{PbCrO_4} \downarrow + 2CH_3COO^-$$



10.

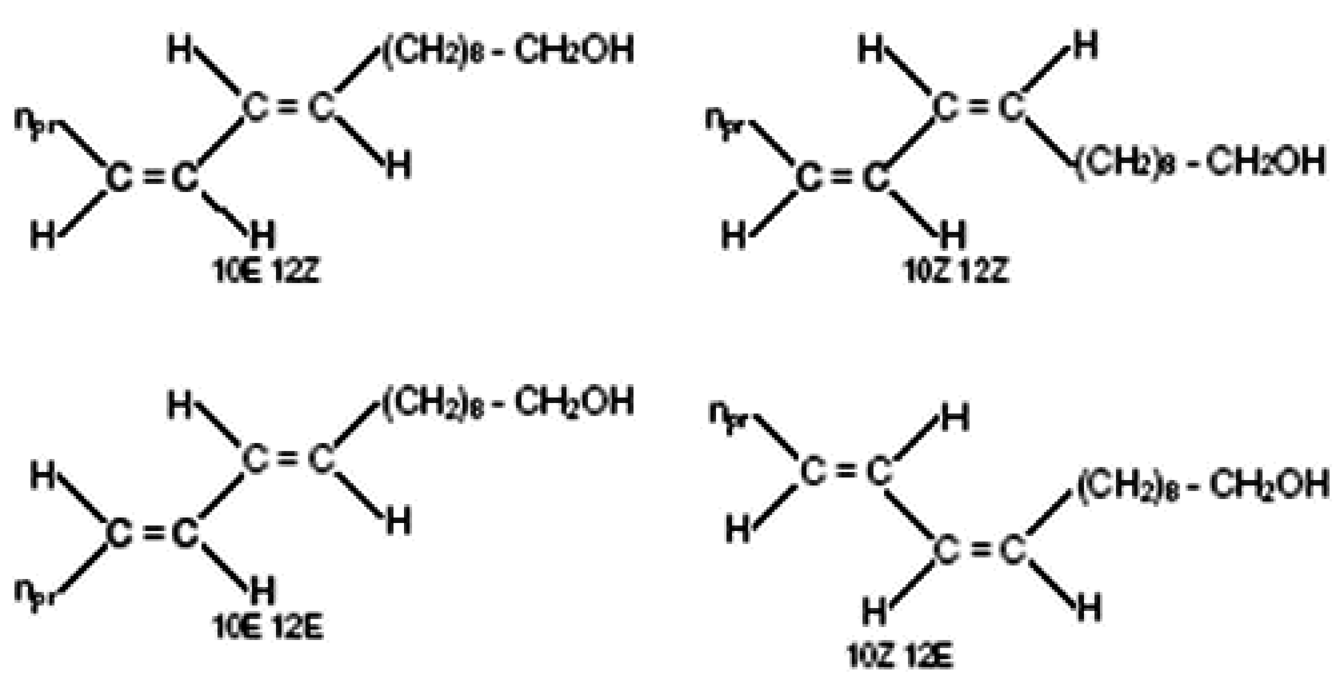


11. No. of double bonds = 2

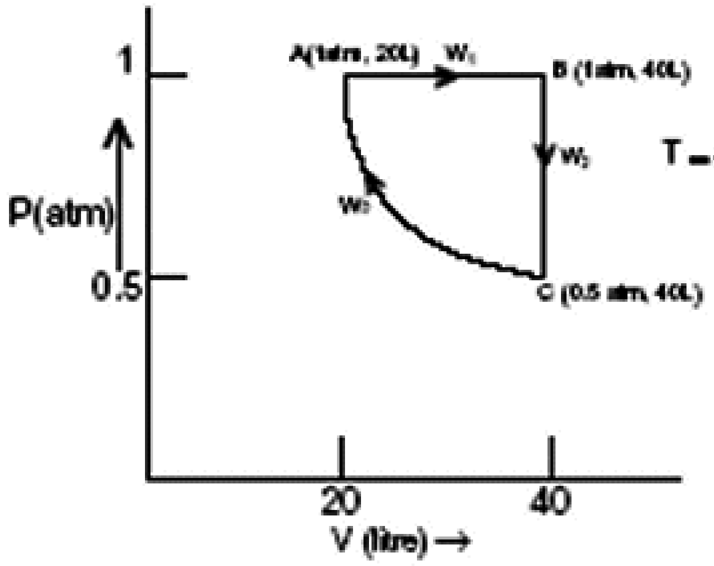
$CH_3CH_2CH_2 - CH = CH - CH = CH - (CH_2)_8 - CH_2 - OH$
(Bombykol)

$CH_3 - CH_2 - CH_2 - CH_2 - CH_2 - CH_2 - CH_2 - (CH_2)_8 - CH_2OH$
(A)

4 Geometrical isomers are possible.



12. (I)



$T = \frac{PV}{nR} = \frac{1 \times 20}{2 \times 0.082} = 121.95K$

(II) Total work (W) = $W_1 + W_2 + W_3$

$= -P\Delta V + 0 + 2.303\ nRT \log\frac{V_1}{V_2}$

$= -1 \times 20 + 2.303 \times 2 \times 0.082 \times 121.95 \log 2$

$= -20 + 13.86 = -6.13\ L\ atm$

Since the system has returned to its initial state i.e. the process is cyclic so $\Delta U = 0$

$\Delta U = q + W = 0$, so

$q = -W = 6.13 L.\ atm = 620.7\ J$

In a cyclic process heat absorbed is completely converted into work

(III) Entropy is a state function and since the system has returned to its initial state so $\Delta S = 0$.
Similarly $\Delta H = 0$ and $\Delta U = 0$ for the same reason i.e. U and H are also state functions having definite viaues in a given state of a system.

Solution Physics

1. (a) 2(Fundamental Frequency of A) = 3 (Fundamental Frequency of B)

$$\frac{2v_A}{2l_A}=\frac{3v_B}{4l_B}$$

As, $v=\sqrt{\frac{\gamma RT}{M}}$, we have $v=\sqrt{\frac{\gamma_A}{M_A}}=\frac{3}{4}\sqrt{\frac{\gamma_B}{M_B}};\frac{M_A}{M_B}=\frac{400}{189}$

(b) $\frac{v_A}{v_B}=\sqrt{\frac{\gamma_A M_B}{\gamma_B M_A}}=\frac{3}{4}$

2. (a) Time between two consecutive collisions = $\frac{2l}{v}=t$

Here, $t=\frac{1}{500}$ sec and l = 1m $\Rightarrow$ v =1000 m/s. Also, $v=\sqrt{\frac{3RT}{M}}\Rightarrow T=160K$

(b) Average kinetic energy of an atom of a monoatomic gas = $\frac{3}{2}kT$

$\therefore E_{av}=\frac{3}{2}kT=3.312\times10^{-21}$ Joules

(c) From gas equation $PV=\frac{m}{M}RT\Rightarrow m=0.3012gm$

3. (a) As the pressure force exerted by liquid A is radial & symmetric its net value is zero.
(b) For equilibrium, Buoyant force = weight of the body

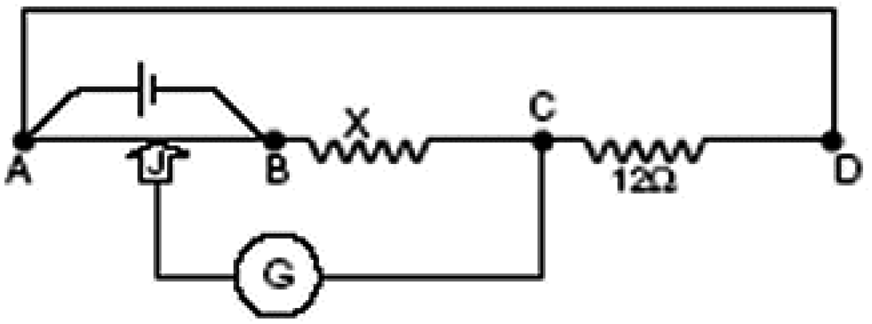
$\Rightarrow h_A\rho_A Ag+h_B\rho_B Ag=(h_A+h+h_B)A\rho cg$ (where ρ_c = density of cylinder)

$$h=\left(\frac{h_A\rho_A+h_B\rho_B}{\rho_C}\right)-(h_A+h_B)=0.25cm$$

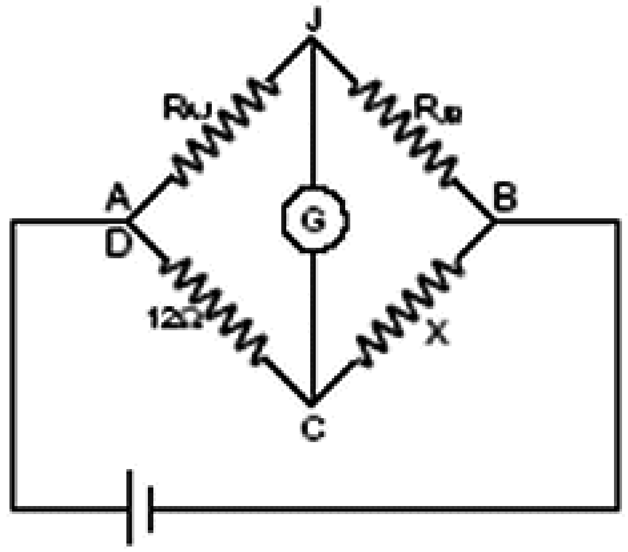
(c) $a=\frac{F'_{Buoyant}-Mg}{M}=\left[\frac{h_A\rho_A+\rho_B(h+h_B)-(h+h_A+h_B)\rho_C}{\rho_C(h+h_A+h_C)}\right]g=\frac{g}{6}$ upwards

4. (a) No

(b)



(c) ∵ Bridge is balanced



$$\frac{R_{AJ}}{R_{JB}}=\frac{0.6\rho}{0.4\rho}=\frac{12\Omega}{X}\Rightarrow X=8\Omega$$

Where ρ is resistance per unit length.

5. (a) If x is the difference in quantum number of two states then ${}^{x+1}C_2 = 6 \Rightarrow x = 3$

Now, we have $\frac{-z^2(13.6eV)}{n^2}=-0.85eV$ ....(1)

and $\frac{-z^2(13.6eV)}{(n+3)^2}=-0.544eV$ ....(2)

Solving (1) and (2) we get n = 12 and z = 3

(b) Smallest wavelength λ is given by $\frac{hc}{\lambda}$ = (0.85 - 0.544) eV

Solving, we get $\lambda \approx 4052nm$.

6. (a) As there is symmetry about the line SP, fringes will be circular.

(b) $\frac{I_{min}}{I_{max}} = \left(\frac{\sqrt{1}-\sqrt{0.36I}}{\sqrt{1}+\sqrt{0.36I}}\right)^2 = \left(\frac{0.4}{1.6}\right)^2 = \frac{1}{16}$

(c) For maximum at P, path difference = $n\lambda$.

If AB is shifted by a distance x, it will cause an additional path difference of 2x.

$2x = \lambda$ (for minimum value of x) $\Rightarrow x = \lambda/2 = 300nm$

7. (a) Torque due to magnetic forces shoould act opposite to that of gravity i.e. along the -ve y-axis. If $M\hat{k}$ is the magnetic moment

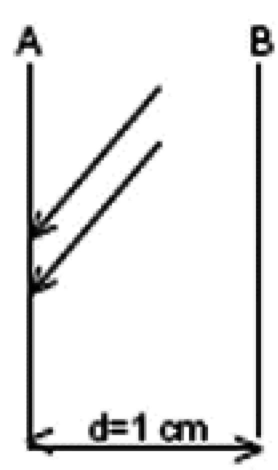
$\vec{\tau}_B = \vec{M}\times\vec{B} = M\hat{k}\times(3\hat{i}+4\hat{k})B_0 = 3MB_0\hat{j} \Rightarrow M \text{ is } -ve$

$\therefore$ I should be clockwise i.e. from P to Q

(b) $\vec{F} = I(\vec{L}\times\vec{B}) \Rightarrow \vec{F}_{RS} = I[(-b\hat{j})\times(3\hat{i}+4\hat{k})B_0] = IB_0b[3\hat{k}-4\hat{i}]$

(c) $3(abI)B_0 = mga/2 \Rightarrow I = \frac{mg}{6B_0b}$

8. (a) Number of photoelectrons emitted from plate A upto t = 10 s



$ne = \frac{(5\times10^{-4})\times10^{16}}{10^{6}}\times10 = 5\times10^{7}$

(b) Charge on plate B at t = 10 sec

$Q_b = 33.7\times10^{-12} - 5\times10^{7}\times1.6\times10^{-19} = 25.7\times10^{-12}$ C

also $Q_a \times 10^{-12}$ C

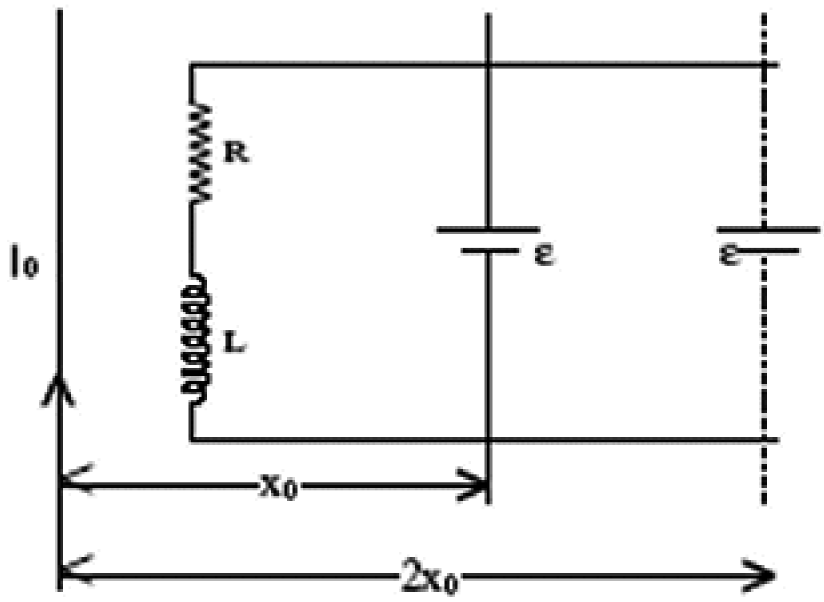
$E = \frac{\sigma_B}{2\varepsilon_0} - \frac{\sigma_A}{2\varepsilon_0} = \frac{1}{2A\varepsilon_0}(Q_B - Q_A)$

$= \frac{17.7\times10^{-12}}{5\times10^{-4}\times8.85\times10^{-12}} = 2000N/C$

(c) K.E. of most energetic particles = $(h\nu - \phi) + e(Ed) = 23eV$

9. (a) $\varepsilon - L\frac{di}{dt} - iR = 0 \Rightarrow \left|\frac{d\phi}{dt}\right| - \frac{Ldi}{dt} = iR$

(b) Net change in flux $= \int d\phi = \int_{x_0}^{2x_0} \frac{\mu_0 I_0}{2\pi x}dx - Li_1 = \frac{\mu_0 I_0}{2\pi}\ln 2 - Li_1$



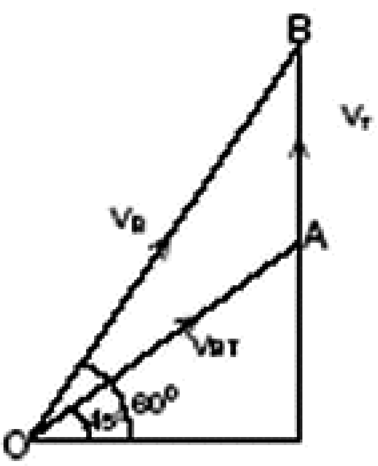
Net charge flown through resistance $R = \frac{\text{total change in flux}}{R} = \frac{\frac{\mu_0}{2\pi}I_0 \ln 2 - Li_1}{R}$

(c) Current in the circuit for $T \le t \le 2T$ is given by $I = I_1 e^{-(t-T)R/L}$

At $t = 2T$, $I = I_1/4 \Rightarrow \frac{L}{R} = \frac{R}{2\ln 2}$

10. (a) From the diagram $\vec{V}_{BT}$ makes an angle of 45^0 with the x-axis.

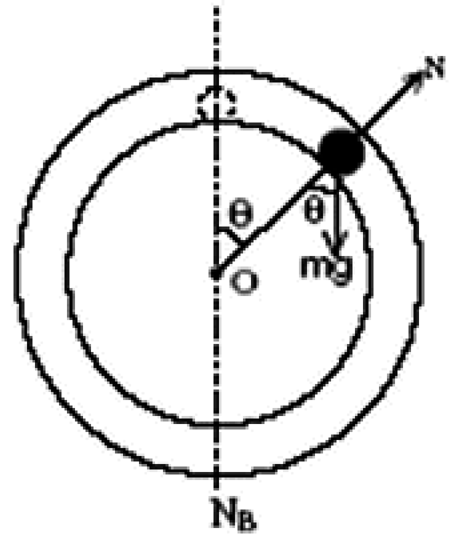
(b) Using sine rule



$$\frac{V_B}{\sin 135^0} = \frac{V_T}{\sin 15^0}$$

$$\Rightarrow V_B = 2 m/s$$

11. (a) From F.B.D. of the ball



$$mg\cos\theta - N = \frac{mv^2}{(R + d/2)} \quad \text{..........(I)}$$

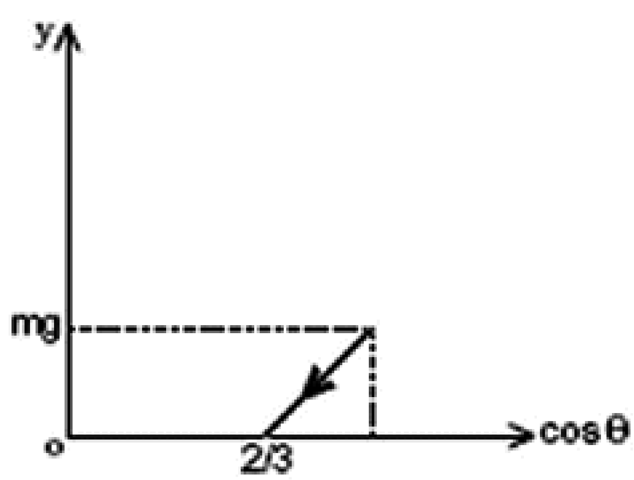
Using conservation of energy

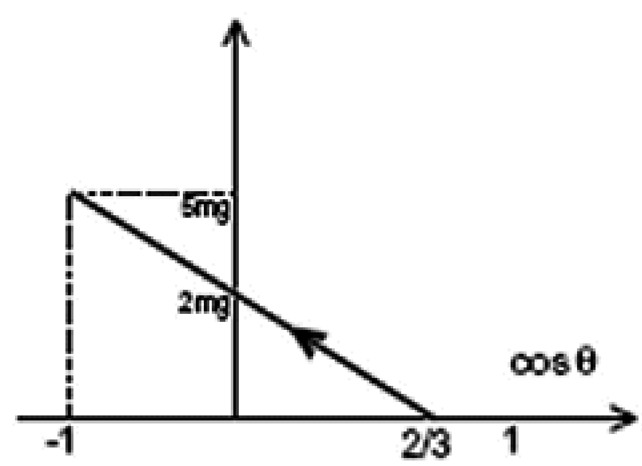
$$mg[R + (d/2)] \quad (1 - \cos\theta) = (½)\, mv^2 \quad \text{.......... (II)}$$

From (I) and (II)

$N = mg\,(3\cos\theta - 2)$ radially outwards

(b)





12. (a) Magnitude of centrifugal force on both B and C is

$$F_1 = m\omega^2 l$$

∴ Resultant force

$$F = 2F_1 \cos 30^0 = \sqrt{3}\, m\omega^2 l$$

(b) Torque due to F about A

$$\tau_A = \frac{F\sqrt{3}\, l}{2} = i\alpha = 2ml^2\alpha \qquad a_{\text{tangential}} = \alpha \frac{l}{\sqrt{3}} \Rightarrow F_x = -\frac{F}{4m}$$

$$F_x + F = \frac{F}{4m} 3m = \frac{3F}{4} \Rightarrow Fx = -\frac{F}{4}$$

$$F_y = \sqrt{3}\, ml\omega^2$$

1. Clearly $A_1 + A_2 = a + b$

$$\frac{1}{H_1}+\frac{1}{H_2}=\frac{1}{a}+\frac{1}{b} \Rightarrow \frac{H_1+H_2}{H_1H_2}=\frac{a+b}{ab}=\frac{A_1+A_2}{G_1G_2}$$

$$\Rightarrow \frac{G_1G_2}{H_1H_2}=\frac{A_1+A_2}{H_1+H_2}$$

Also $\frac{1}{H_1}=\frac{1}{a}+\frac{1}{3}\left(\frac{1}{b}-\frac{1}{a}\right) \Rightarrow H_1=\frac{3ab}{2a+b}$

and $\frac{1}{H_2}=\frac{1}{a}+\frac{2}{3}\left(\frac{1}{b}-\frac{1}{a}\right) \Rightarrow H_2=\frac{3ab}{2a+a}$

$$\Rightarrow \frac{A_1+A_2}{H_1+H_2}=\frac{a+b}{3ab\left(\frac{1}{2b+a}+\frac{1}{2a+b}\right)}=\frac{(2b+a)\,(2a+b)}{9ab}$$

2. Let $P(n) : (25)^{n+1} - 24\,n + 5735$

For $n = 1$, $P(1) : 625 - 24 + 5735 = 6336 = (24)^2 \times (11)$

which is divisible by 24^2

Hence $P(1)$ is true

Let $P(k)$ be the true , where $k \geq 1$

$\Rightarrow (25)^{k+1} - 24\,k + 5735 = (24)^2\,\lambda$ where $\lambda \in N$

For $n = k+1$

$P(k+1) : (25)^{k+2} - 24\,k + 5735$

$= 25[(25)^{k+1} - 24\,k + 5735] + 25\,.\,24\,.\,k - (25)\,(5735) + 5735 - 24\,(k+1)$

$= 25((24)^2\lambda + (24)^2 k - 5735 \times 24 - 24$

$= 25((24)^2\lambda + (24)^2 k - (24)\,(5736)$

$= 25((24)^2\lambda + (24)^2 k - (24)^2\,(239)$

$= (24)^2\,[25\lambda + k - 230]$

which is divisible by $(24)^2$

Hence, by the method of mathematical induction result is true $\forall n \in N$

3. For $x > 0$ $\cot^{-1}x = \sin^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right)$ and

for x< 0 $\cot^{-1} x = \pi - \sin^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right)$

In both the cases $\sin(\cot^{-1}x) = \left(\frac{1}{\sqrt{x^2+1}}\right)$

$$\Rightarrow \cos\left(\tan^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \cos\left(\cos^{-1}\sqrt{\frac{x^2+1}{x^2+2}}\right) = \sqrt{\frac{x^2+1}{x^2+2}}$$

4. E_1 : coin in fair, E_2 : coin is biased, A : first toss shows head and second toss shows tail

$$P(E_1/A) = \frac{P(A/E_1)P(E_1)}{P(A/E_1)P(E_1) + P(A/E_2)P(E_2)}$$

$$= \frac{\frac{m}{N}\cdot\frac{1}{2}\cdot\frac{1}{2}}{\frac{m}{N}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{N-m}{N}\cdot\frac{2}{3}\cdot\frac{1}{3}} = \frac{9m}{9n+8N-8m} = \frac{9m}{8N+m}$$

5. The given equation can be written as $(z^p - 1)(z^p - 1) = 0$

$$\Rightarrow \alpha = e^{i\frac{2k_1\pi}{p}} \text{ or } e^{i\frac{2k_2\pi}{q}}$$

since p and q are prime to each other

$\Rightarrow$ L.C.M. of $\frac{2k_1\pi}{p}$ and $\frac{2k_2\pi}{q}$ is equal to $2m\pi$, where m is the L.C.M of k_1 and k_2 but $\alpha \neq 1$.

Hence there is no value of α which satisfy both the equations simultaneously other than 1.
So α will be the root of either $z^p - 1 = 0$ or $z^q - 1 = 0$

$\Rightarrow$ either $1 + \alpha + \ldots\ldots \alpha^{p-1} = 0$ or $1 + \alpha + \ldots. + \alpha^{q-1} = 0$

6. The line y = mx meets the given lines in $P\left(\frac{1}{m+1}, \frac{m}{m+1}\right)$ and $Q\left(\frac{3}{m+1}, \frac{3m}{m+1}\right)$

Hence equation of L_1 is $y - \frac{m}{m+1} = 2\left(x - \frac{1}{m+1}\right) \Rightarrow y - 2x - 1 = \frac{-3}{m+1}$ ....(1)

and that of L_1 is $y - \frac{3m}{m+1} = -3\left(x - \frac{3}{m+1}\right) \Rightarrow y + 3x - 3 = \frac{6}{m+1}$ ....(2)

From (1) and (2) $\frac{y-2x-1}{y+3x-3} = -\frac{1}{2}$

$\Rightarrow x = 3y + 5 = 0$ which is a straight line

7. Let the equation of the line by (y-2) = m(x-8) where m < 0

$\Rightarrow P = \left(8 - \frac{2}{m}, 0\right)$ and $Q = (0, 2 - 8m)$

Now OP + OQ = $\left|8 - \frac{2}{m}\right| + |2 - 8m| = 10 + \frac{2}{-m} + 8(-m) \geq 10 + 2\sqrt{\frac{2}{-m} \times (-m)} \geq 18$

8. Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and O be the centre

Tangent at $P(x_1, y_1)$ is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1=0$ whose slope $= -\frac{b^2x_1}{a^2y_1}$

Focus is $S(ae,o)$

Equation of the line perpendicular to tangent at P is $y=\frac{a^2y_1}{b^2x_1}(x-ae)$ ..... (1)

Equation of OP is $y=\frac{y_1}{x_1}x$.......(2)

(1) and (2) intersect

$\Rightarrow \frac{y_1}{x_1}x=\frac{a^2y_1}{b_2x_1}(x-ae)$

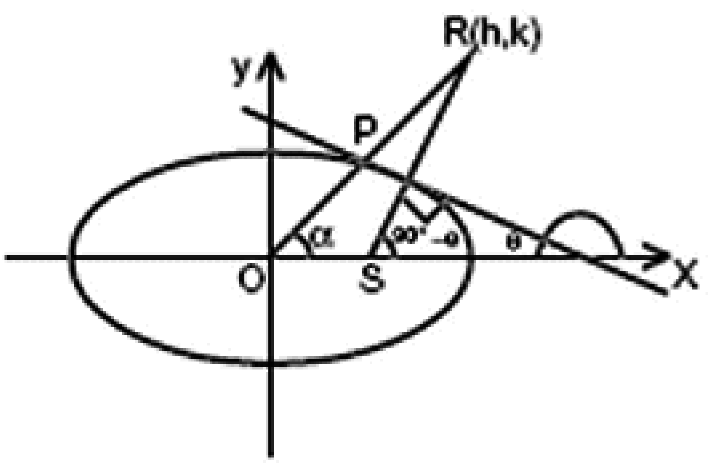
$\Rightarrow x(a^2-b^2)=a^3e$

$\Rightarrow x.a^2e^2=a^3e \Rightarrow x=a/e$

which is the corresponding directrix.

Alternate

$P=(a\cos\beta,\ b\sin\beta), R=(h,k)$



$\tan\alpha=\frac{b}{a}\tan\beta=\frac{k}{h}$ ....(1)

and slope of tangent

$\tan(\pi-\theta)=-\frac{b}{a}\cot\beta$

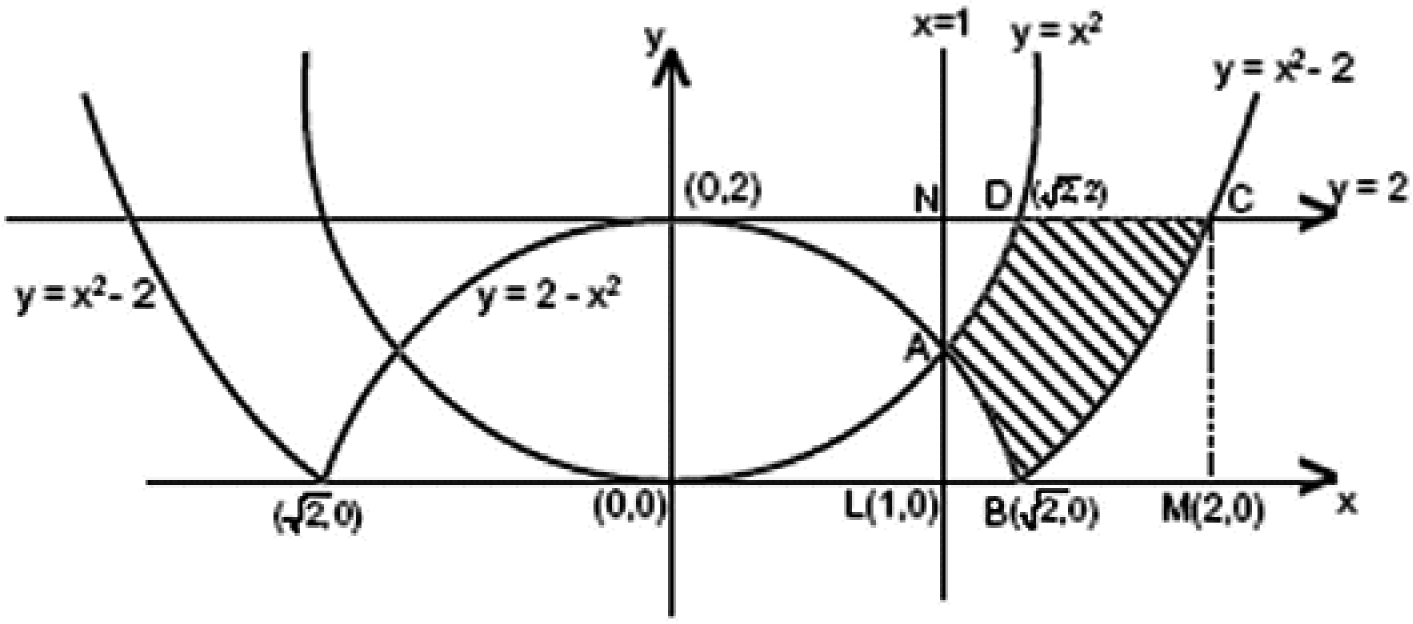
$\therefore \tan(90^0-\theta)=\frac{k}{h\pm ae}=\frac{a}{b}\tan\beta=\frac{a}{b}.\frac{a}{b}.\frac{k}{h}$ (from(1))

or $\frac{k}{h\pm ae}=\frac{a^2k}{b^2h}$

$\Rightarrow h=\pm\frac{a}{e}$ i.e. R lies on the corresponding directrix, when $s=(ae,o), h=\frac{a}{e}$

and when $s=(-ae,o), h=-\frac{a}{e}$

9. Required area is ABCD



$$\int_1^{\sqrt{2}} [x^2-(2-x^2)]dx + 2(2-\sqrt{2}) - \int_{\sqrt{2}}^{2}(x^2-2)dx = \left|\frac{2x^3}{3}-2x\right|_1^{\sqrt{2}} + 2(2-\sqrt{2}) - \left|\frac{x^3}{3}-2x\right|_{\sqrt{2}}^{2}$$

$$= \frac{2}{3}(2\sqrt{2}-1) - 2(\sqrt{2}-1) + 2\sqrt{2}(\sqrt{2}-1) - \left\{\left(\frac{8}{3}-4\right) - \left(\frac{2\sqrt{2}}{3}-2\sqrt{2}\right)\right\}$$

$$= \frac{4\sqrt{2}}{3} - \frac{2}{3} + 6 - 4\sqrt{2} + \frac{4}{3} + \frac{2\sqrt{2}}{3} - 2\sqrt{2} = \frac{20}{3} + 2\sqrt{2} - 4\sqrt{2} - 2\sqrt{2} = \frac{20}{3} - 4\sqrt{2} \text{ sq units}$$

Alternate

Required area = area of rectangle LMCN - 2 (area of LBA) - area of BMC

$$= 2 - \int_1^{\sqrt{2}}(2-x^2)dx - \int_{\sqrt{2}}^{2}(x^2-2)\,dx$$

$$= \frac{20}{3} - 4\sqrt{2} \text{ sq units}$$

10. Here $V = |\vec{a}.(\vec{b}\times\vec{c})| = \left\| \begin{matrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{matrix} \right\|$

Also $L = \dfrac{a_1+b_1+c_1+a_2+b_2+c_2+a_3+b_3+c_3}{3}$

$\geq [(a_1+b_1+c_1)(a_2+b_2+c_2)(a_3+b_3+c_3)]^{1/2}$ $\quad (\because AM \geq GM)$

$\Rightarrow L^3 \geq (a_1+b_1+c_1)(a_2+b_2+c_3)(a_3+b_3+c_3)$

$\geq |(a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2) - (a_1b_3c_2 + a_2b_1c_3 + a_3b_2c_1)| \Rightarrow L^3 \geq V.$

Alternate

$$V = \left\| \begin{matrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{matrix} \right\| = |(a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2) - (a_1b_3c_2 + a_2b_1c_3 + a_3b_2c_1)|$$

$V \leq \max.\{(a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2), \ (a_1b_3c_2 + a_2b_1c_3 + a_3b_2c_1)\}$

Let $(a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2) \geq (a_1b_3c_2 + a_2b_1c_3 + a_3b_3c_1)$

$V \leq (a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2)$

$$L = \left(\frac{a_1 + b_2 + c_3}{3}\right) + \left(\frac{a_2 + b_3 + c_1}{3}\right) + \left(\frac{a_3 + b_1 + c_2}{3}\right)$$

$L \geq (a1b2c3)^{1/3} + (a_2b_3c_1)^{1/3} + (a_3b_1c_2)^{1/3}$

$L^3 \geq (a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2)$........ (2)

From equation (1) and (2),

$V \leq L^3$.

similarly we can prove for the case $(a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2) < (a_1b_3c_2 + a_2b_1c_3 + a_3b_2c_1)$

11. $I = \int (x^{3m} + x^{2m} + x^m)(2x^{2m} + 3x^m + 6)^{1/m} dx,\ x > 0$

$\int (x^{3m-1} + x^{2m-1} + x^m)(2x^{2m} + 3x^{2m} + 6x^m)^{1/m} dx$

Let $2x^{3n} + 3x^{2m} + 6x^m = t \Rightarrow (x^{2m-1} + x^{2m-1} + x^{m-1})dx = \frac{dt}{6m}$

$$\Rightarrow \int t^{1/m} \cdot \frac{dt}{6m} = \frac{1}{6m}\left(\frac{t^{(m+1)/m}}{\left(\frac{m+1}{m}\right)}\right) + c = \frac{(2x^{3m} + 3x^{2m} + 6x^m)^{(m+1)/m}}{6(m+1)} + c$$

12. $$g\ (f\ (x)) = \begin{cases} x + a + 1, & x < -a \\ (x + a - 1)^2 + b, & -a \leq x < 0 \\ x^2 + b, & 0 \leq x < 1 \\ (x - 2)^2 + b, & x \geq 1 \end{cases}$$

$$\left.\begin{aligned} g(f(-a)^-) &= 1 \\ g(f(-a)^+) &= 1 + b \end{aligned}\right\} \Rightarrow b = 0 \text{ (as gof is continuous everywhere)}$$

$$\text{Now, } \left.\begin{aligned} g(f(0)^-) &= (a-1)^2 \\ g(f(0)^+) &= 0 \end{aligned}\right\} \Rightarrow a = 1$$

For this value of a and b, g (f(x)) is continuous.

$$\text{And } g(f(x)) = \begin{cases} x + 2, & x < -1 \\ x^2, & -1 \leq x < 1 \\ (x-2)^2, & x \geq 1 \end{cases}$$

Clearly at x =0 g(f(x)) is differnetiable.

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