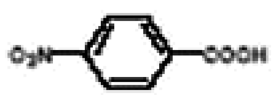
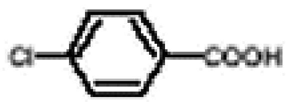
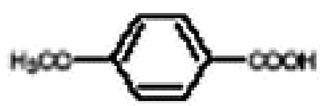
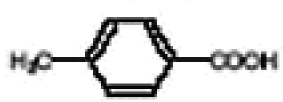
Chemistry

Time: 2 hours **Marks: 60**

[10 × 2 = 20]

1. Calculate the molarity of water if its density is 1000 kg/m^3.
2. The average velocity of gas molecules is 400 m/sec. Calculate its rms velocity at the same temperature.
3. Write down the heterogeneous catalyst involved in the polymerisation of ethylene.
4. Which one is more soluble in diethyl ether anhydrous $AlCl_3$ or hydrous $AlCl_3$? Explain in terms of bonding.
5. Using VSEPR theory, draw the shape of PCl_5 and BrF_5.
6. A racemic mixture of (±) 2–phenyl propanoic acid on esterification with (+) 2–butanol gives two esters. Mention the stereochemistry of the two esters produced.
7. Wavelength of high energy transition of H-atoms is 91.2nm. Calculate the corresponding wavelength of He atoms.
8. Match the K_a values

		K_a
a)	Benzoic acid	3.3×10^{-5}
b)		6.3×10^{-5}
c)		30.6×10^{-5}
d)		6.4×10^{-5}
e)		4.2×10^{-5}

9. Write down reactions involved in the extraction of Pb. What is the oxidation number of lead in litharge?

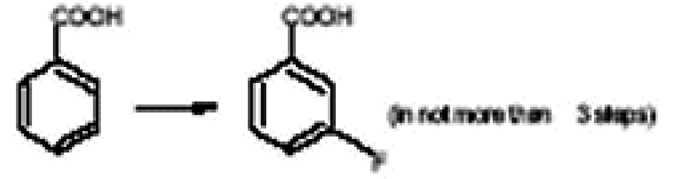
10. Following two amino acids lionise and glutamine form dipeptide linkage. What are two possible dipeptides?

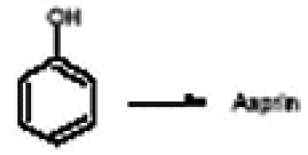
[10 × 4 = 40]

11. a) You are given marbles of diameter 10 mm. They are to be placed such that their centres are lying in a square bound by four lines each of length 40 mm. What will be the arrangements of marbles in a plane so that maximum number of marbles can be placed inside the area? Sketch the diagram and derive expression for the number of molecules per unit area.
    b) 1 gm of charcoal adsorbs 100 ml 0.5 M CH_3COOH to form a monolayer, and thereby the molarity of CH_3COOH reduces to 0.49. Calculate the surface area of the charcoal adsorbed by each molecule of acetic acid. Surface area of charcoal = 3.01×10^2 m^2/gm.

12. a) Will the pH of water be same at 4°C and 25°C? Explain.
    b) Two students use same stock solution of $ZnSO_4$ and a solution of $CuSO_4$. The emf of one cell is 0.03 V higher than the other. The conc. of $CuSO_4$ in the cell with higher emf value is 0.5 M. Find out the conc. of $CuSO_4$ in the other cell (2.203 RT/F = 0.06).

13. Convert

a) 

b) 

14. There is a solution of p–hydroxy benzoic acid and p–amino benzoic acid. Discuss one method by which we can separate them and also write down the confirmatory tests of the functional groups present.

15. $A(C_6H_{12}) \xrightarrow{HCl} \underset{(C_6H_{13}Cl)}{B} + C$

$B \xrightarrow{alc.\ KOH} D$ (isomer of A)

$D \xrightarrow{ozonolysis} E$ (it gives negative test with Fehling solution but responds to iodoform test).

$A \xrightarrow{Ozonolysis} F + G$ (both give positive Tollen's test but do not give iodoform test).

$F + G \xrightarrow{conc.\ NaOH} HCOONa +$ a primary alcohol

Identify from A to G.

16. Identify the following:

$Na_2CO_3 \xrightarrow{SO_2} A \xrightarrow{Na_2CO_3} B \xrightarrow[\Delta]{Elemental\ S} C \xrightarrow{I_2} D$

Also mention the oxidation state of S in all the compounds.

17. Write the IUPAC nomenclature of the given complex along with its hybridisation and structure.

$K_2[Cr(NO)(NH_3)(CN)_4]$, $\mu = 1.73$ BM

18. A mixture consists A (yellow solid) and B (colourless solid) which gives lilac colour in flame.

a) Mixture gives black precipitate C on passing $H_2S_{(g)}$.

b) C is soluble in aqua–regia and on evaporation of aqua–regia and adding $SnCl_2$ gives greyish black precipitate D.

The salt solution with NH_4OH gives a brown precipitate.

i) The sodium extract of the salt with $CCl_4/FeCl_3$ gives a violet layer.

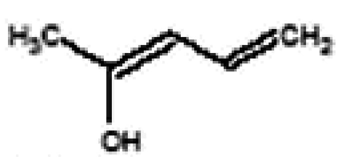
ii) The sodium extract gives yellow precipitate with $AgNO_3$ solution which is insoluble in NH_3.

Identify A and B, and the precipitates C and D.

19. a) Match the following if the molecular weights of X, Y and Z are same.

	Boiling Point	K_b
X	100	0.68
Y	27	0.53
Z	253	0.98

b) C_v value of He is always 3R/2 but C_v value of H_2 is 3R/2 at low temperature and 5R/2 at moderate temperature and more than 5R/2 at higher temperature explain in two to three lines.

20. a) 

Write resonance structure of the given compound.

b) Compound A of molecular formula $C_9H_7O_2Cl$ exists in ketoform and predominantly in enolic form 'B'. On oxidation with $KMnO_4$, 'A' gives m-chlorobenzoic acid.

Identify 'A' and 'B'.

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MATHEMATICS

Time: 2 hours *Marks: 60*

1. If z_1 and z_2 are two complex numbers such that $|z_1| < 1 < |z_2|$ then prove that $\left|\frac{1-z_1\bar{z}_2}{z_1-z_2}\right|<1$. [2]

2. Find a point on the curve $x^2 + 2y^2 = 6$ whose distance from the line $x + y = 7$, is minimum. [2]

3. If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c are real positive numbers, $abc = 1$ and $A^TA = I$, then find the value of $a^3 + b^3 + c^3$. [2]

4. Prove that $2^k\binom{n}{0}\binom{n}{k}-2^{k-1}\binom{n}{1}\binom{n-1}{k-1}+2^{k-2}\binom{n}{2}\binom{n-2}{k-2}-\ldots(-1)^k\binom{n}{k}\binom{n-k}{0}=\binom{n}{k}$. [2]

5. If f is an even function then prove that $\int_0^{\pi/2} f(\cos 2x)\cos x\,dx=\sqrt{2}\int_0^{\pi/4} f(\sin 2x)\cos x\,dx$. [2]

6. For a student to qualify, he must pass at least two out of three exams. The probability that he will pass the 1st exam is p. If he fails in one of the exams then the probability of his passing in the next exam is $\frac{p}{2}$ otherwise it remains the same. Find the probability that he will qualify. [2]

7. For the circle $x^2 + y^2 = r^2$, find the value of r for which the area enclosed by the tangents drawn from the point P(6, 8) to the circle and the chord of contact is maximum. [2]

8. Prove that there exists no complex number z such that $|z| < \frac{1}{3}$ and $\sum_{r=1}^{n} a_r z^r = 1$ where $|a_r| < 2$. [2]

9. A is targeting to B, B and C are targeting to A. Probability of hitting the target by A, B and C are $\frac{2}{3}, \frac{1}{2}$ and $\frac{1}{3}$ respectively. If A is hit then find the probability that B hits the target and C does not. [2]

10. If a function $f : [-2a, 2a] \to R$ is an odd function such that $f(x) = f(2a - x)$ for $x \in [a, 2a]$ and the left hand derivative at $x = a$ is 0 then find the left hand derivative at $x = -a$. [2]

11. Using the relation $2(1 - \cos x) < x^2, x \neq 0$ or otherwise, prove that $\sin(\tan x) \ge x, \forall x \in \left[0, \frac{\pi}{4}\right]$ [4]

12. If a, b, c are in A.P., a^2, b^2, c^2 are in H.P., then prove that either $a = b = c$ or a, b, $-\frac{c}{2}$ form a G.P.

[4]

13. If $x^2 + (a - b)x + (1 - a - b) = 0$ where $a, b \in R$ then find the values of a for which equation has unequal real roots for all values of b. [4]

14. Normals are drawn from the point P with slopes m_1, m_2, m_3 to the parabola $y^2 = 4x$. If locus of P with $m_1 m_2 = \alpha$ is a part of the parabola itself then find α. [4]

15. If the function $f : [0, 4] \to R$ is differentiable then show that

(i). For $a, b \in (0, 4)$, $(f(4))^2 - (f(0))^2 = 8f'(a)\, f(b)$

(ii). $\int_0^4 f(t)\,dt = 2\left[\alpha f(\alpha^2) + \beta f(\beta^2)\right]$ $\forall\ 0 < \alpha, \beta < 2$ [4]

16. (i). Find the equation of the plane passing through the points (2, 1, 0), (5, 0, 1) and (4, 1, 1).

(ii). If P is the point (2, 1, 6) then find the point Q such that PQ is perpendicular to the plane in (i) and the mid point of PQ lies on it. [4]

17. If $P(1) = 0$ and $\frac{dP(x)}{dx} > P(x)$ for all $x \geq 1$ then prove that $P(x) > 0$ for all $x > 1$. [4]

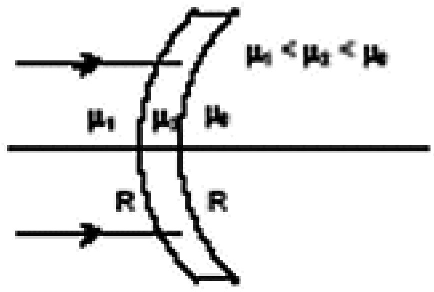
18. If I_n is the area of n sided regular polygon inscribed in a circle of unit radius and O_n be the area of the polygon circumscribing the given circle, prove that $I_n = \frac{O_n}{2}\left(1 + \sqrt{1 - \left(\frac{2I_n}{n}\right)^2}\right)$. [4]

19. If $\vec{u}, \vec{v}, \vec{w}$ are three non-coplanar unit vectors and α, β, γ are the angles between $\vec{u}$ and $\vec{v}$, $\vec{v}$ and $\vec{w}$, $\vec{w}$ and $\vec{u}$ respectively and $\vec{x}, \vec{y}, \vec{z}$ are unit vectors along the bisectors of the angles α, β, γ respectively. Prove that $\left[\vec{x}\times\vec{y}\ \ \vec{y}\times\vec{z}\ \ \vec{z}\times\vec{x}\right] = \frac{1}{16}\left[\vec{u}\ \ \vec{v}\ \ \vec{w}\right]^2 \sec^2\frac{\alpha}{2}\sec^2\frac{\beta}{2}\sec^2\frac{\gamma}{2}$. [4]

20. A right circular cone with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionality constant = k > 0). Find the time after which the cone is empty. [4]

1. If n^{th} division of main scale coincides with $(n+1)^{th}$ divisions of vernier scale. Given one main scale division is equal to 'a' units. Find the least count of the vernier.

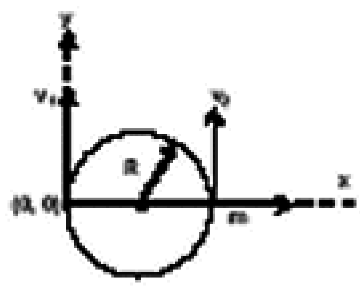
2. Find the focal length of the lens shown in the figure. The radii of curvature of both the surfaces are equal to R.



3. Frequency of a photon emitted due to transition of electron of a certain element from L to K shell is found to be 4.2×10^{18} Hz. Using Moseley's law, find the atomic number of the element, given that the Rydberg's constant $R = 1.1 \times 10^7\ m^{-1}$.

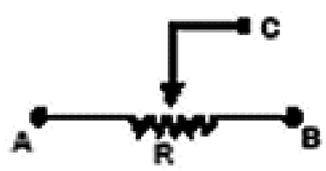
4. An insulated container containing mono atomic gas of molar mass m is moving with a velocity v_0. If the container is suddenly stopped, find the change in temperature.

5. A particle of mass m, moving in a circular path of radius R with a constant speed v_2 is located at point (2R, 0) at time t = 0 and a man starts moving with a velocity v_1 along the +ve y-axis from origin at time t = 0. Calculate the linear momentum of the particle w.r.t. the man as a function of time.

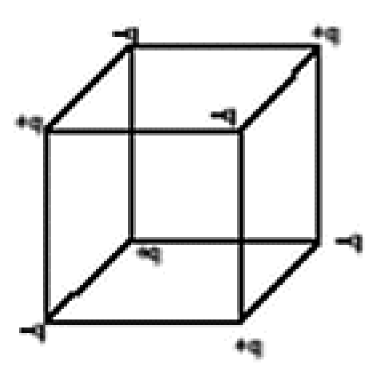


6. A tuning fork of frequency 480 Hz resonates with a tube closed at one end of length 16 cm and diameter 5 cm in fundamental mode. Calculate velocity of sound in air.

7. How a battery is to be connected so that the shown rheostat will behave like a potential divider? Also indicate the points about which output can be taken.



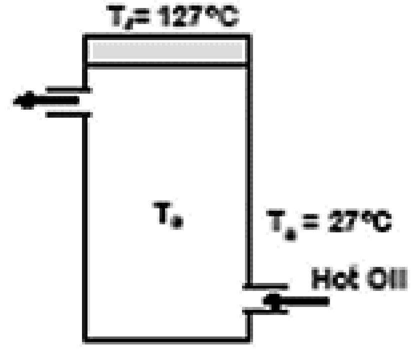
8. Charges +q and −q are located at the corners of a cube of side a as shown in the figure. Find the work done to separate the charges to infinite distance.



9. A radioactive sample emits n β-particles in 2 sec. In next 2 sec it emits 0.75 n β-particle, what is the mean life of the sample?

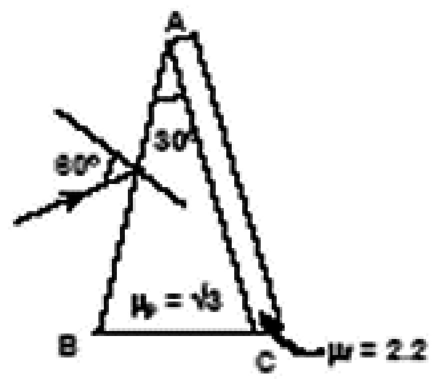
10. In a photoelectric experiment set up, photons of energy 5 eV falls on the cathode having work function 3 eV. (a) If the saturation current is $i_A = 4 \mu A$ for intensity 10^{-5} W/m^2, then plot a graph between anode potential and current. (b) Also draw a graph for intensity of incident radiation $2 \times 10^{-5}\ W/m^2$.

11. Hot oil is circulated through an insulated container with a wooden lid at the top whose conductivity K = 0.149 J/(m-°C-sec), thickness t = 5 mm, emissivity = 0.6. Temperature of the top of the lid is maintained at T_ℓ = 127°. If the ambient temperature T_a= 27°C. Calculate
(a) rate of heat loss per unit area due to radiation from the lid.
(b) temperature of the oil. (Given $\sigma = \frac{17}{3} \times 10^{-8}$)



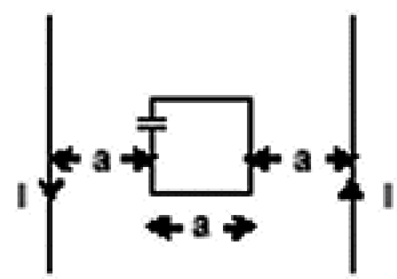
12. Two masses m_1 and m_2 connected by a light spring of natural length ℓ_0 is compressed completely and tied by a string. This system while moving with a velocity v_0 along +ve x-axis pass through the origin at t= 0. At this position the string snaps. Position of mass m_1 at time t is given by the equation
$x_1(t) = v_0t - A(1 - \cos\omega t)$
Calculate
(a) position of the particle m_2 as a function of time.
(b) ℓ_0 in terms of A.

13. Shown in the figure is a prism of an angle 30° and refractive index $\mu_p = \sqrt{3}$. Face AC of the prism is covered with a thin film of refractive index μ_f = 2.2. A monochromatic light of wavelength λ = 550 nm fall on the face AB at an angle of incidence of 60°. Calculate
(a) angle of emergence.
(b) minimum value of thickness t so that intensity of emergent ray is maximum.



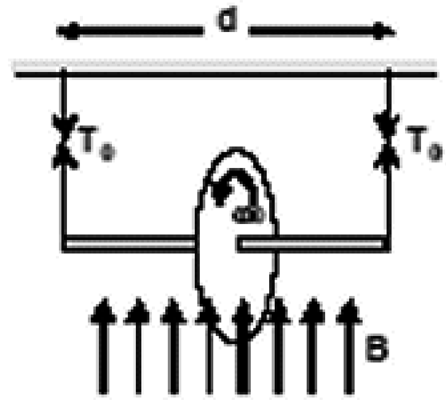
14. A body is projected vertically upwards from the bottom of a crater of moon of depth $\frac{R}{100}$ where R is the radius of moon with a velocity equal to the escape velocity on the surface of moon. Calculate maximum height attained by the body from the surface of the moon.

15. A square loop of side 'a' with a capacitor of capacitance C is located between two current carrying long parallel wires as shown. The value of I in the wires is given as I = b sinωt.
(a) Calculate maximum current in the square loop.
(b) Draw a graph between charges on the upper plate of the capacitor vs time.



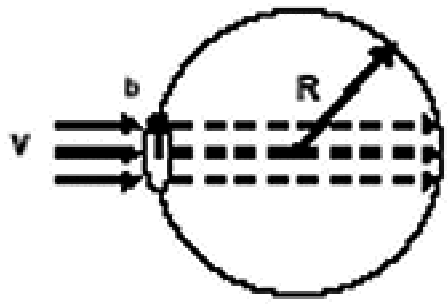
16. A charge +Q is fixed at the origin of the co-ordinate system while a small electric dipole of dipole-moment $\vec{P}$ pointing away from the charge along the x-axis is set free from a point far away from the origin.
(a) Calculate the K.E. of the dipole when it reaches to a point (d, 0).
(b) Calculate the force on the charge +Q at this moment.

17. A wheel of radius R having charge Q, uniformly distributed on the rim of the wheel is free to rotate about a light horizontal rod. The rod is suspended by light inextensible strings and a magnetic field B is applied as shown in the figure. The initial tensions in the strings are T_0. If the breaking tension of the strings are $\frac{3T_0}{2}$, find the maximum angular velocity ω_0 with which the wheel can be rotated.



18. A string tied between x = 0 and x = ℓ vibrates in fundamental mode. The amplitude A, tension T and mass per unit length μ is given. Find the total energy of the string.

19. A bubble having surface tension T and radius R is formed on a ring of radius b (b << R). Air is blown inside the tube with velocity v as shown. The air molecule collides perpendicularly with the wall of the bubble and stops. Calculate the radius at which the bubble separates from the ring.



20. Shown in the figure is a container whose top and bottom diameters are D and d respectively. At the bottom of the container, there is a capillary tube of outer radius b and inner radius a. The volume flow rate in the capillary is Q. If the capillary is removed the liquid comes out with a velocity of v_0. The density of the liquid is given as ρ. Calculate the coefficient of viscosity η.