SOLUTIONS TO IIT–JEE 2004 (SCREENING)

MEMORY BASED QUESTIONS

MATHEMATICS

1. The angle between the tangents drawn from the point (1, 4) to the parabola $y^2 = 4x$ is

(a) $\pi/2$ (b) $\pi/3$

(c) $\pi/4$ (d) $\pi/6$

Sol. **b**

Any tangent to $y^2 = 4x$ is $y = mx + \frac{1}{m}$. Since it passes through (1, 4), we have $4 = m + \frac{1}{m}$

$\Rightarrow \quad m^2 - 4m + 1 = 0$

$\Rightarrow \quad m_1 + m_2 = 4,\ m_1 . m_2 = 1$

$\Rightarrow \quad |m_1 - m_2| = 2\sqrt{3}$

If θ is the required angle, then $\tan\theta = \frac{2\sqrt{3}}{1+1} = \sqrt{3} \Rightarrow \theta = \pi/3$

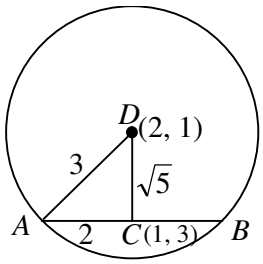
2. The radius of the circle, having centre at (2, 1), whose one of the chord is a diameter of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$

(a) 1 (b) 2

(c) 3 (d) $\sqrt{3}$

Sol. **c**

The centre C of the given circle is (1, 3) and radius is 2. So AB, a diameter of the given circle has its mid point as (1, 3). The radius DA of the required circle = 3.



3. If $\sin\left(\cot^{-1}(x+1)\right) = \cos\left(\tan^{-1} x\right)$, then $x =$

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 0 (d) $\frac{9}{4}$

Sol. **a**

$$\sin\left(\cot^{-1}(x+1)\right) = \sin\left(\sin^{-1}\frac{1}{\sqrt{x^2+2x+2}}\right) = \frac{1}{\sqrt{x^2+2x+2}}$$

$$\cos\left(\tan^{-1} x\right) = \cos\left(\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right) = \frac{1}{\sqrt{1+x^2}}$$

Thus $\dfrac{1}{\sqrt{x^2+2x+2}} = \dfrac{1}{\sqrt{1+x^2}} \Rightarrow x^2 + 2x + 2 = x^2 + 1 \Rightarrow x = -\dfrac{1}{2}$

4. $\displaystyle\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx =$

(a) $\dfrac{\pi}{2}+1$ (b) $\dfrac{\pi}{2}-1$

(c) π (d) 1

Sol. **b**

Let $x = \cos 2\theta$, $0 \le 2\theta \le \dfrac{\pi}{2}$.

Now $\sqrt{\dfrac{1-x}{1+x}} = \sqrt{\dfrac{1-\cos 2\theta}{1+\cos 2\theta}} \Rightarrow \displaystyle\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx = 4\int_0^{\pi/4} \sin^2\theta\, d\theta = 2\int_0^{\pi/4} (1-\cos 2\theta)\,d\theta$

$$= 2\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/4} = 2\left[\frac{\pi}{4} - \frac{1}{2}\right] = \frac{\pi}{2} - 1$$

5. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then $\alpha =$

(a) ± 3 (b) ± 2

(c) ± 5 (d) 0

Sol. **a**

$125 = |A^3| = |A|^3 \Rightarrow |A| = 5 \Rightarrow \alpha^2 - 4 = 5$

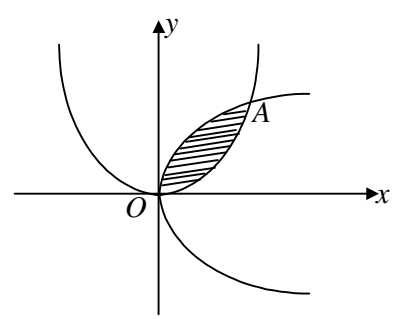
$\Rightarrow \quad \alpha = \pm 3$

6. If the area bounded by $y = ax^2$ and $x = ay^2$, $a > 0$, is 1, then $a =$

(a) 1 (b) $\dfrac{1}{\sqrt{3}}$

(c) $\dfrac{1}{3}$ (d) $-\dfrac{1}{\sqrt{3}}$

Sol. **b**



The x coordinate of A is $\dfrac{1}{a}$.

According to the given condition

$$1 = \int_0^{1/a} \left(\sqrt{\frac{x}{a}} - ax^2\right) dx$$

$$= \frac{1}{\sqrt{a}} \cdot \frac{2}{3}\left[x^{3/2}\right]_0^{1/a} - \frac{a}{3}\left[x^3\right]_0^{1/a}$$

$\Rightarrow \quad a = \dfrac{1}{\sqrt{3}}$

7. ${}^{n-1}C_r = (K^2 - 3) \,.\, {}^{n}C_{r+1}$, if $K \in$

(a) $\left[-\sqrt{3}, \sqrt{3}\right]$ (b) $(-\infty, -2)$

(c) $(2, \infty)$ (d) $(\sqrt{3}, 2]$

Sol. **d**

We have $\dfrac{(n-1)!}{r!\,(n-r-1)!} = (K^2 - 3)\dfrac{n!}{(r+1)!(n-r-1)!}$, $0 \le r \le n-1$

$\Rightarrow \quad 1 = (K^2 - 3)\dfrac{n}{r+1}$

$\Rightarrow \quad K^2 = \dfrac{r+1}{n} + 3, \ \dfrac{1}{n} \le \dfrac{r+1}{n} \le 1$

$\Rightarrow \quad K^2 \in \left[\dfrac{1}{n} + 3, 4\right], \ n \ge 2$

$\Rightarrow \quad K \in \left[-2, -\sqrt{\dfrac{1}{n} + 3}\right] \cup \left[\sqrt{\dfrac{1}{n} + 3}, 2\right], \quad n \ge 2$

Thus option (d) is correct.

8. The first term of an infinite geometric progression is x and its sum is 5. Then

(a) $0 \le x \le 10$ (b) $0 < x < 10$

(c) $-10 < x < 0$ (d) $x > 10$

Sol. **b**

According to the given conditions

$5 = \dfrac{x}{1-r}$, r being the common ratio

$\Rightarrow \quad r = 1 - \dfrac{x}{5}$

As $\quad |r| < 1, \ \left|1 - \dfrac{x}{5}\right| < 1$

$\Rightarrow \quad -1 < \dfrac{x}{5} - 1 < 1$

$\Rightarrow \quad 0 < x < 10$

9. If θ and ϕ are acute angles satisfying $\sin\theta = \dfrac{1}{2}$, $\cos\phi = \dfrac{1}{3}$, then $\theta + \phi \in$

(a) $\left(\dfrac{\pi}{3}, \dfrac{\pi}{2}\right]$ (b) $\left(\dfrac{\pi}{2}, \dfrac{2\pi}{3}\right)$

(c) $\left(\dfrac{2\pi}{3}, \dfrac{5\pi}{6}\right)$ (d) $\left(\dfrac{5\pi}{6}, \pi\right)$

Sol. **b**

$\sin\theta = \dfrac{1}{2} \qquad \Rightarrow \theta = \dfrac{\pi}{6}$

$$\cos\phi = \frac{1}{3} \qquad \Rightarrow \frac{\pi}{3} < \phi < \frac{\pi}{2}$$

Thus $\frac{\pi}{2} < \theta + \phi < \frac{2\pi}{3}$

10. If $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x,\ y(0) = 1$, then $y\left(\frac{\pi}{2}\right) =$

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

Sol. **c**

The given differential equation is

$$\frac{\cos x}{2+\sin x}dx + \frac{dy}{y+1} = 0$$

$\Rightarrow \quad \ln(2+\sin x) + \ln(y+1) = \ln c$

$\Rightarrow \quad (y+1)(2+\sin x) = c$

$\Rightarrow \quad 2 \times 2 = c \ \Rightarrow c = 4.$

Thus $y+1 = \frac{4}{2+\sin x}$

$\Rightarrow \quad y = \frac{2-\sin x}{2+\sin x}$

$\Rightarrow \quad y\left(\frac{\pi}{2}\right) = \frac{1}{3}$

11. If one root of the equation $x^2 + px + q = 0$ is the square of the other, then

(a) $p^3 + q^2 - q(3p+1) = 0$ (b) $p^3 + q^2 + q(1+3p) = 0$

(c) $p^3 + q^2 + q(3p-1) = 0$ (d) $p^3 + q^2 + q(1-3p) = 0$

Sol. **d**

Let α and α^2 be the roots, then $\alpha + \alpha^2 = -p$, $\alpha^3 = q$.

Now $(\alpha + \alpha^2)^3 = \alpha^3 + \alpha^6 + 3\alpha^3(\alpha + \alpha^2)$

$\Rightarrow \quad -p^3 = q + q^2 - 3pq$

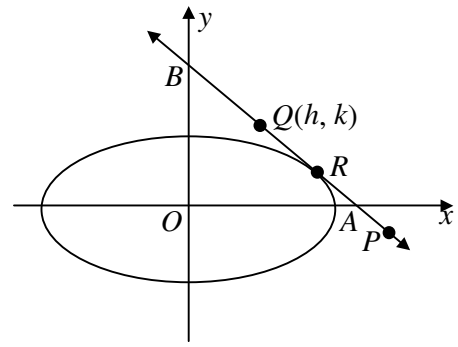
$\Rightarrow \quad p^3 + q^2 + q(1-3p) = 0$

12. The locus of the middle point of the intercept of the tangents drawn from an external point to the ellipse $x^2 + 2y^2 = 2$, between the coordinates axes, is

(a) $\frac{1}{x^2} + \frac{1}{2y^2} = 1$ (b) $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$

(c) $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$ (d) $\frac{1}{2x^2} + \frac{1}{y^2} = 1$

Sol. **c**



Let the point of contact be $R \equiv (\sqrt{2}\cos\theta, \sin\theta)$.

Equation of tangent AB is $\frac{x}{\sqrt{2}}\cos\theta + y\sin\theta = 1$.

$\Rightarrow \quad A \equiv (\sqrt{2}\sec\theta, 0), B \equiv (0, \text{cosec}\theta)$.

Let the middle point Q of AB be (h, k).

$\Rightarrow \quad h = \frac{\sec\theta}{\sqrt{2}}, k = \frac{\text{cosec}\theta}{2}$

$\Rightarrow \quad \cos\theta = \frac{1}{h\sqrt{2}}, \sin\theta = \frac{1}{2k}$

$\Rightarrow \quad \frac{1}{2h^2} + \frac{1}{4k^2} = 1$.

Thus required locus is $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$.

13. ω is an imaginary cube root of unity. If $(1 + \omega^2)^m = (1 + \omega^4)^m$, then least positive integral value of m is

(a) 6 (b) 5

(c) 4 (d) 3

Sol. **d**

The given equation reduces to $(-\omega)^m = (-\omega^2)^m$

$\Rightarrow \quad \omega^m = 1 \Rightarrow m = 3$

14. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then $\vec{b} =$

(a) $\hat{i}$ (b) $\hat{i} - \hat{j} + \hat{k}$

(c) $2\hat{j} - \hat{k}$ (d) $2\hat{i}$

Sol. **a**

Let $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Now $\hat{j} - \hat{k} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ b_1 & b_2 & b_3 \end{vmatrix}$

$\Rightarrow \quad b_3 - b_2 = 0, \quad b_1 - b_3 = 1, b_2 - b_1 = -1$

$\Rightarrow \quad b_3 = b_2, b_1 = b_2 + 1$

Now $\vec{a}.\vec{b} = 1 \quad \Rightarrow b_1 + b_2 + b_3 = 1 \quad \Rightarrow 3b_2 + 1 = 1 \Rightarrow b_2 = 0$

$\Rightarrow \quad b_1 = 1, b_3 = 0$

Thus $\vec{b} = \hat{i}$

15. The ratio of the sides of a triangle ABC is $1 : \sqrt{3} : 2$. The ratio $A : B : C$ is

(a) 3 : 5 : 2 (b) $1 : \sqrt{3} : 2$

(c) 3 : 2 : 1 (d) 1 : 2 : 3

Sol. **d**

According to the given condition

$$a = \lambda,\ b = \sqrt{3}\,\lambda,\ c = 2\lambda, \text{ for some } \lambda \in R.$$

Now $\cos A = \dfrac{3\lambda^2 + 4\lambda^2 - \lambda^2}{4\sqrt{3}\lambda^2} = \dfrac{6}{4\sqrt{3}} = \dfrac{\sqrt{3}}{2} \Rightarrow A = \dfrac{\pi}{6}$

$$\cos B = \frac{\lambda^2 + 4\lambda^2 - 3\lambda^2}{4\lambda^2} = \frac{2\lambda^2}{4\lambda^2} = \frac{1}{2} \Rightarrow B = \frac{\pi}{3}$$

Thus $C = \dfrac{\pi}{2}$

Therefore $A : B : C = 1 : 2 : 3$.

16. If $\int_0^{t^2} xf(x)dx = \dfrac{2}{5}t^5$, $t > 0$, then $f\left(\dfrac{4}{25}\right) =$

(a) $\dfrac{2}{5}$ (b) $\dfrac{5}{2}$

(c) $-\dfrac{2}{5}$ (d) 1

Sol. **a**

Given that $\int_0^{t^2} x\ f(x)dx = \dfrac{2}{5}t^5$

$\Rightarrow \quad 2t\,.\,t^2 f(t^2) = 2t^4$

$\Rightarrow \quad f(t^2) = t$

$\Rightarrow f\left(\dfrac{4}{25}\right) = \dfrac{2}{5}$, as $t > 0$

17. Let $f(x) = \sin x + \cos x,\ g(x) = x^2 - 1$. Thus $g(f(x))$ is invertible for $x \in$

(a) $\left[-\dfrac{\pi}{2}, 0\right]$ (b) $\left[-\dfrac{\pi}{2}, \pi\right]$

(c) $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$ (d) $\left[0, \dfrac{\pi}{2}\right]$

Sol. **c**

$g(f(x)) = g(\sin x + \cos x) = \sin 2x$, which is invertible for $x \in \left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$

18. The area bounded by the angle bisectors of the lines $x^2 - y^2 + 2y = 1$ and the line $x + y = 3$, is

(a) 2 (b) 3

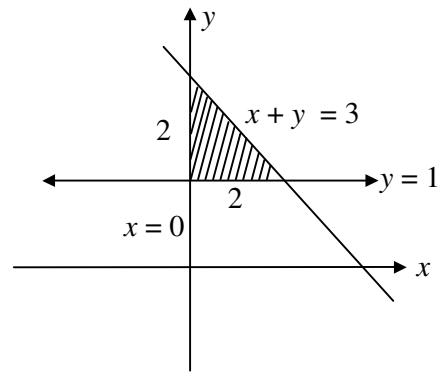
(c) 4 (d) 6

Sol. **a**

The angle bisectors of the lines given by

$x^2 - y^2 + 2y = 1$ are $x = 0$, $y = 1$.

The required area = $\frac{1}{2} \times 2 \times 2 = 2$



19. If $x^2 + 2ax + 10 - 3a > 0$ for all $x \in R$, then

(a) $-5 < a < 2$ (b) $a < -5$

(c) $a > 5$ (d) $2 < a < 5$

Sol. **a**

According to the given condition

$4a^2 - 4(10 - 3a) < 0$

$\Rightarrow$ $a^2 + 3a - 10 < 0$

$\Rightarrow$ $(a + 5)(a - 2) < 0$

$\Rightarrow$ $-5 < a < 2$

20. Let $f(x) = x^3 + bx^2 + cx + d$, $0 < b^2 < c$. Then f

(a) is bounded (b) has a local maxima

(c) has a local minima (d) is strictly increasing

Sol. **d**

$f'(x) = 3x^2 + 2bx + c$

Now its discriminant $= 4(b^2 - 3c)$

$= 4(b^2 - c) - 8c < 0$, as $b^2 < c$ and $c > 0$.

Therefore $f'(x) > 0$ for all $x \in R$.

Hence f is strictly increasing.

21. The value of λ for which the system of equations $2x - y - z = 12$, $x - 2y + z = -4$, $x + y + \lambda z = 4$ has no solution is

(a) 3 (b) –3

(c) 2 (d) –2

Sol. **d**

The coefficient determinant $= \begin{vmatrix} 2 & -1 & -1 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix}$

$= -3\lambda - 6$

For no solution the necessary condition is $-3\lambda - 6 = 0$

$\Rightarrow$ $\lambda = -2$

It can be seen that for $\lambda = -2$, there is no solution for the given system of equations.

22. Let $f(x)=\begin{cases} x^{\alpha}\ln x, x>0 \\ 0, \quad x=0 \end{cases}$. Rolle's theorem is applicable to f for $x \in [0, 1]$, if $\alpha =$

(a) –2 (b) –1

(c) 0 (d) $\frac{1}{2}$

Sol. **d**

For Rolle's theorem to be applicable to f, for $x \in [0, 1]$, we should have (i) $f(1) = f(0)$, (ii) f is continuous for $x \in [0, 1]$ and f is differentiable for $x \in (0, 1)$

From (i) $f(1) = 0$, which is true.

From (ii) $0 = f(0) = f(0+) = \lim_{x\to 0} x^{\alpha}.\ln x$, which is true only for positive values of α. Thus (d) is correct.

23. The line $2x + \sqrt{6}\,y = 2$ is a tangent to the curve $x^2 - 2y^2 = 4$. The point of contact is

(a) $(4, -\sqrt{6})$ (b) $(7, -2\sqrt{6})$

(c) (2, 3) (d) $(\sqrt{6}, 1)$

Sol. **a**

Solving the line and the curve we get $x = 4$ and $y = -\sqrt{6}$.

Thus point of contact is $(4, -\sqrt{6})$.

24. Three distinct numbers are selected from first 100 natural numbers. The probability that all the three numbers are divisible by 2 and 3 is

(a) $\frac{4}{25}$ (b) $\frac{4}{35}$

(c) $\frac{4}{55}$ (d) $\frac{4}{1155}$

Sol. **d**

The numbers should be divisible by 6. Thus the number of favourable ways is ${}^{16}C_3$ (as there are 16 numbers in first 100 natural numbers, divisible by 6). Required probability is $\frac{{}^{16}C_3}{{}^{100}C_3} = \frac{16\times 15\times 14}{100\times 99\times 98} = \frac{4}{1155}$.

25. If f is a strictly increasing function, then $\lim_{x\to 0}\frac{f(x^2)-f(x)}{f(x)-f(0)}$ is equal to

(a) 0 (b) 1

(c) –1 (d) 2

Sol. **c**

$$\lim_{x\to 0}\frac{f(x^2)-f(x)}{f(x)-f(0)} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x\to 0}\frac{2xf'(x^2)-f'(x)}{f'(x)} \quad \text{(using L' Hospitals's rule)}$$

$$= -1+\lim_{x\to 0}\frac{2xf'(x^2)}{f'(x)} = -1, f'(0) \neq 0, \text{ as } f \text{ is strictly increasing.}$$

26. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then $k =$

(a) $\frac{2}{9}$ (b) $\frac{9}{2}$

(c) 0 (d) –1

Sol. **b**

Any point on $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ is

$(2\lambda + 1, 3\lambda - 1, 4\lambda + 1), \lambda \in R.$

Any point on $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ is

$(\mu + 3, 2\mu + k, \mu), \mu \in R.$

The given lines intersect if and only if the system of equations (in λ and μ)

$2\lambda + 1 = \mu + 3$ (1)

$3\lambda - 1 = 2\mu + k$ (2)

$4\lambda + 1 = \mu$ (3)

has a unique solution.

Solving (1) and (3), we get $\lambda = -\frac{3}{2}$, $\mu = -5$.

From (2), we get $-\frac{9}{2} - 1 = -10 + k \Rightarrow k = \frac{9}{2}$.

27. If $\ln(x + y) = 2xy$, then $y'(0) =$

(a) 1 (b) –1

(c) 2 (d) 0

Sol. **a**

$\ln(x + y) = 2xy.$

$\Rightarrow \quad \left(1 + \frac{dy}{dx}\right)/(x + y) = 2\left(x.\frac{dy}{dx} + y\right)$

$\Rightarrow \quad \frac{dy}{dx} = \frac{1 - 2xy - 2y^2}{2x^2 + 2xy - 1}$

$\Rightarrow \quad y'(0) = \frac{1-2}{-1} = 1$, as at $x = 0$, $y = 1$.

28. A unit vector in the plane of the vectors $2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \hat{j} + \hat{k}$ and orthogonal to $5\hat{i} + 2\hat{j} + 6\hat{k}$ is

(a) $\frac{6\hat{i} - 5\hat{k}}{\sqrt{61}}$ (b) $\frac{3\hat{j} - \hat{k}}{\sqrt{10}}$

(c) $\frac{2\hat{i} - 5\hat{j}}{\sqrt{29}}$ (d) $\frac{2\hat{i} + \hat{j} - 2\hat{k}}{3}$

Sol. **b**

Let a unit vector in the plane of $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ be

$$\hat{a}=\alpha\left(2\hat{i}+\hat{j}+\hat{k}\right)+\beta\left(\hat{i}-\hat{j}+\hat{k}\right)$$

$$=(2\alpha+\beta)\hat{i}+(\alpha-\beta)\hat{j}+(\alpha+\beta)\hat{k}$$

As $\hat{a}$ is unit vector, we have

$$(2\alpha+\beta)^2+(\alpha-\beta)^2+(\alpha+\beta)^2=1$$

$$\Rightarrow \quad 6\alpha^2+4\alpha\beta+3\beta^2=1 \qquad (1)$$

As $\hat{a}$ is orthogonal to $5\hat{i}+2\hat{j}+6\hat{k}$, we get

$$5(2\alpha+\beta)+2(\alpha-\beta)+6(\alpha+\beta)=0$$

$$\Rightarrow \quad 18\alpha+9\beta=0 \quad \Rightarrow \quad \beta=-2\alpha.$$

From (1), we get $6\alpha^2-8\alpha^2+12\alpha^2=1$

$$\Rightarrow \quad \alpha=\pm\frac{1}{\sqrt{10}} \quad \Rightarrow \quad \beta=\mu\frac{2}{\sqrt{10}}$$

Thus $\hat{a}=\pm\left(\frac{3}{\sqrt{10}}\hat{j}-\frac{1}{\sqrt{10}}\hat{k}\right)$

Note : *All these questions of IIT–JEE Screening 2004 are based on the memory of the select PIE students who appeared in the examination. PIE Education does not take any responsibility for any sort of discrepancy whatsoever.*