IIT JEE-2009 PAPER-2
SECTION - I
Single Correct Choice Type

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE** is correct.

Q.20 A line with positive direction cosines passes through the point P(2, – 1, 2) and makes equal angles with the coordinate axes. The line meets the plane

$$2x + y + z = 9$$

at point Q. The length of the line segment PQ equals

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

[**Sol.** **[C]**

$l^2 + l^2 + l^2 = 1 \Rightarrow \quad l = \dfrac{1}{\sqrt{3}}$

equation is $\dfrac{x-2}{1/\sqrt{3}} = \dfrac{y+1}{1/\sqrt{3}} = \dfrac{z-2}{1/\sqrt{3}} = k$

it cuts 2x + y + z = 9 at Q

$\therefore \quad 2\left(2+\dfrac{k}{\sqrt{3}}\right)+\left(-1+\dfrac{k}{\sqrt{3}}\right)+\left(2+\dfrac{k}{\sqrt{3}}\right)=9 \quad \Rightarrow \quad \dfrac{4k}{\sqrt{3}} = 4 \Rightarrow k = \sqrt{3}$

$\therefore \quad Q = (3, 0, 3)$

$\therefore \quad PQ = \sqrt{1+1+1} = \sqrt{3}$ **Ans.**]

Q.21 If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is

(A) $\dfrac{n(4n^2-1)c^2}{6}$ (B) $\dfrac{n(4n^2+1)c^2}{3}$ (C) $\dfrac{n(4n^2-1)c^2}{3}$ (D) $\dfrac{n(4n^2+1)c^2}{6}$

[**Sol.** **[C]**

$\because \quad \dfrac{n}{2}[2a + (n-1)d] = cn^2 \quad \forall\ n$

$\Rightarrow \quad an + n(n-1)\dfrac{d}{2} = cn^2 \Rightarrow \quad n^2\dfrac{d}{2} + n\left(a - \dfrac{d}{2}\right) = cn^2$

$\therefore \quad d = 2c$ and $a - \dfrac{d}{2} = 0 \Rightarrow \quad a = \dfrac{d}{2} = c$

$\therefore$ A.P. is c, 3c, 5c, ...

$\therefore$ sum of squares $= c^2 + (3c)^2 + (5c)^2 + ...$

$$= c^2\sum(2n-1)^2 = c^2\sum(4n^2-4n+1)$$

$$= c^2\left\{\frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n\right\}$$

$$= \frac{nc^2}{6}(8n^2 + 12n + 4 - 12n - 12 + 6)$$

$$= \frac{nc^2}{6}(8n^2 - 2) = \frac{nc^2(4n^2-1)}{3}$$ **Ans.**]

Q.22 The locus of the orthocentre of the triangle formed by the lines

$(1 + p)x - py + p(1 + p) = 0,$

$(1 + q)x - qy + q(1 + q) = 0$

and $y = 0$, where $p \neq q$, is

(A) a hyperbola (B) a parabola (C) an ellipse (D) a straight line

[**Sol.** **[D]**

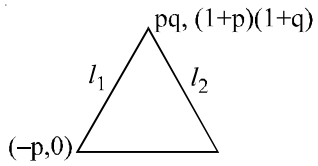
$(1 + p)x - py + p(1 + p) = 0$

$(1 + q)x - qy + q(1 + q) = 0$

$q(1 + p)x - pqy + pq(1 + p) = 0$

$p(1 + q)x - pqy + pq(1 + q) = 0$

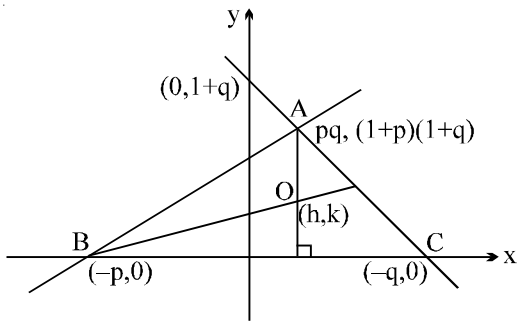
$x(q - p) + pq(p - q) = 0$



$x = pq; \qquad y = \dfrac{(1+p)\cancel{p}q + \cancel{p}(1+p)}{\cancel{p}}$

$y = (1 + p)(1 + q)$

$h = pq$



$m_{AC} = \dfrac{(1+q)}{q}$

$m_{OB} \times m_{AC} = -1$

$\Rightarrow \quad \dfrac{k}{h+p} = \dfrac{-q}{1+q}$

$\dfrac{kq}{h(1+q)} = \dfrac{-q}{1+q}; \qquad x + y = 0 \qquad \Rightarrow$ **(D)**]

Q.23 The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x-axis at Q. If M is the mid point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the point

(A) $\left(\pm\dfrac{3\sqrt{5}}{2}, \pm\dfrac{2}{7}\right)$ (B) $\left(\pm\dfrac{3\sqrt{5}}{2}, \pm\dfrac{\sqrt{19}}{4}\right)$ (C) $\left(\pm 2\sqrt{3}, \pm\dfrac{1}{7}\right)$ (D) $\left(\pm 2\sqrt{3}, \pm\dfrac{4\sqrt{3}}{7}\right)$

[**Sol.** **[C]**

Let P on ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{4} = 1$ is $(4\cos\theta, 2\sin\theta)$

equation of normal is

$4x \sec\theta - 2y \operatorname{cosec}\theta = 16 - 4$

$= 12$

point $Q = (3\cos\theta, 0)$

$\therefore \quad \text{midpoint} = \left(\dfrac{7}{2}\cos\theta, \sin\theta\right) \approx (h, k)$

$h = \dfrac{7}{2}\cos\theta$ and $k = \sin\theta$

$\therefore \quad$ locus is $\dfrac{4h^2}{49} + k^2 = 1 \qquad \Rightarrow \qquad \dfrac{4x^2}{49} + y^2 = 1$

equation of latus rectum is, $x = \pm 0e = \pm \frac{4\sqrt{3}}{2} = \pm 2\sqrt{3}$

$\therefore \quad \frac{4}{49} \times 12 + y^2 = 1 \quad \Rightarrow \quad y^2 = \frac{1}{49} \quad \Rightarrow \quad y = \pm \frac{1}{7}$

$\therefore$ points are $= \left(\pm 2\sqrt{3}, \pm \frac{1}{7}\right)$ **Ans.**]

SECTION - II

Multiple Correct Choice Type

This section contains 5 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/are correct.

Q.24 An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then

(A) Equation of ellipse is $x^2 + 2y^2 = 2$ (B) The foci of ellipse are $(\pm 1, 0)$

(C) Equation of ellipse is $x^2 + 2y^2 = 4$ (D) The foci of ellipse are $(\pm \sqrt{2}, 0)$

[**Sol.** **[A, B]**

$e_E = e$; $e_H = \frac{1}{e}$

$\frac{x^2}{1/2} - \frac{y^2}{1/2} = 1$

$e^2 = 1 + 1$ $S_1S_2 = (\pm 1, 0)$

$e_H = \sqrt{2}$ $ae = 1$

$e_E = \frac{1}{\sqrt{2}}$ $a = \sqrt{2}$

$\frac{x^2}{2} + \frac{y^2}{1} = 1$ $b^2 = a^2(1 - e^2) = 2 \times \frac{1}{2} \Rightarrow b^2 = 1$

$x^2 + 2y^2 = 2 \quad \Rightarrow$ **A, B**]

Q.25 For $0 < \theta < \frac{\pi}{2}$, the solution(s) of $\sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right) \operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$ is (are)

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) $\frac{5\pi}{12}$

[**Sol.** **[C, D]**

$$\frac{\sin\left(\theta + \frac{\pi}{4} - \theta\right)}{\sin\theta \cdot \sin\left(\theta + \frac{\pi}{4}\right)} + \frac{\sin\left(\theta + \frac{\pi}{2} - \left(\theta + \frac{\pi}{4}\right)\right)}{\sin\left(\theta + \frac{\pi}{4}\right)\sin\left(\theta + \frac{\pi}{2}\right)}$$

$$\sin\left(\theta + \frac{5\pi}{4}\right)\sin\left(\theta + \frac{6\pi}{4}\right)$$

$$\cot\theta - \cot\left(\theta+\frac{\pi}{4}\right) + \cot\left(\theta+\frac{\pi}{4}\right) - \cot\left(\theta+\frac{\pi}{2}\right)$$

$$\sqrt{2}\left(\cot\theta - \cot\left(\theta+\frac{3\pi}{2}\right)\right) = 4\sqrt{2} \quad \Rightarrow \quad \cot\theta + \tan\theta = 4$$

$$\tan^2\theta - 4\tan\theta + 1 = 0 \Rightarrow \quad \tan\theta = \frac{4\pm\sqrt{12}}{2} = 2\pm\sqrt{3} \quad \Rightarrow \quad \textbf{C, D}\ ]$$

Q.26 If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x}dx,\ n = 0,1,2,.....,$ then

(A) $I_n = I_{n+2}$ (B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$ (C) $\sum_{m=1}^{10} I_{2m} = 0$ (D) $I_n = I_{n+1}$

[**Sol.** **[A, B, C]**

$$I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x}dx,\ n = 0,1,2,.....,$$

$$I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{\sin x(1+\pi^{-x})}dx$$

$$2I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{\sin x}dx$$

$$I_n = \int_{0}^{\pi} \frac{\sin nx}{\sin x}dx \qquad ....(1) \qquad \left[\begin{array}{l}\pi \text{ if } n \text{ is odd}\\ 0 \text{ if } n \text{ is even}\end{array}\right.$$

$$I_{n+2} = \int_{0}^{\pi} \frac{\sin(n+2)x}{\sin x}dx$$

$$I_{n+2} - I_n = \int_{0}^{\pi} \frac{2\cos(n+1)x\cdot\sin x}{\sin x}dx = \frac{2}{n+1}\sin(n+1)x\Big|_0^{\pi}$$

$$I_{n+2} = I_n \quad \Rightarrow \quad \textbf{A, B, C}\ ]$$

Q.27 For the function $f(x) = x\cos\frac{1}{x},\ x \ge 1,$

(A) for at least one x in the interval $[1, \infty)$, $f(x+2) - f(x) < 2$

(B) $\lim_{x\to\infty} f'(x) = 1$

(C) for all x in the interval $[1, \infty)$, $f(x+2) - f(x) > 2$

(D) $f'(x)$ is strictly decreasing in the interval $[1, \infty)$

[**Sol.** **[B, C, D]**

$f(x) = x \cdot \cos\frac{1}{x} \qquad x \geq 1$

$f'(x) = \cos\frac{1}{x} - x\sin\frac{1}{x} \times \left(-\frac{1}{x^2}\right)$

$= \cos\frac{1}{x} + \frac{1}{x}\sin\frac{1}{x} > 0$ [inc function in $[1, \infty)$]

$f''(x) = -\frac{1}{x^3}\cos\frac{1}{x} - \frac{1}{x^3}\sin\frac{1}{x} - \sin\frac{1}{x}\cdot\left(-\frac{1}{x^2}\right)$

= – ve hence concave down

slope of chord > slope of tangent

$\frac{f(x+2)-f(x)}{(x+2)-x} > m$

but slope of tangent is decreasing function of x with $\lim_{x\to\infty} f(x) = 1 \quad \Rightarrow$ **(C)**

$\frac{f(x+2)-f(x)}{x+2-x} > 1; \quad \frac{f(x+2)-f(x)}{2} > 1$

$\lim_{x\to\infty}\left(\cos\frac{1}{x} + \frac{1}{x}\sin\frac{1}{x}\right) = \lim_{x\to\infty} f'(x) = 1 \quad \Rightarrow$ **(B)**

$f'' < 0$ in $[1, \infty) \quad \Rightarrow \quad f'(x)$ is decreasing $\quad \Rightarrow$ **(D)**]

Q.28 The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose

(A) vertex is $\left(\frac{2a}{3}, 0\right)$ (B) directrix is $x = 0$

(C) latus rectum is $\frac{2a}{3}$ (D) focus is (a, 0)

[**Sol.** **[A, D]**

$P(at^2, 2at)$

$y - 2at = \frac{1}{t}(x - at^2)$

$ty - 2at^2 = x - at^2$

Tangent $\Rightarrow$ $x - ty + at^2 = 0$ meet axis at $T(-at^2, 0)$

Normal $\Rightarrow$ $tx + y - 2at - at^3 = 0$ meets axis at $N(2a + at^2, 0)$

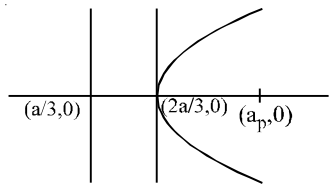
centroid of PTN $\left(\frac{2a+at^2}{3}, \frac{2at}{3}\right) = (x, y) \qquad t = \frac{3y}{2a}$

$\frac{2a + a\left(\frac{3y}{2a}\right)^2}{3} = x \quad \Rightarrow \quad \frac{8a^2 + 9y^2}{12a} = x$

$\Rightarrow \quad 9y^2 = 12a - 8a^2$

$\Rightarrow \quad y^2 = \frac{49}{9}(3x - 29)$

$= \frac{49}{3}\left(x - \frac{29}{3}\right) \qquad \left(\frac{29}{3}, 0\right)$]

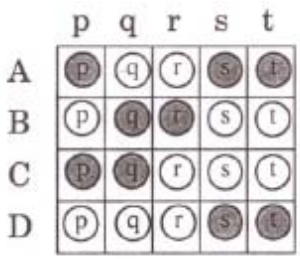


SECTION - III

Matrix - Match Type

This section contains 2 questions. Each question contains statements given in two columns, which have to be matched. The statements in Column I are labelled A, B, C and D, while the statements in Column II are labelled p, q, r, s and t. Any given statement in Column I can have correct matching with ONE OR MORE statement(s) in Column II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:

If the correct matches are A - p, s and t; B - q and r; C - p and q; and D - s and t; then the correct darkening of bubbles will look like the following.



Q.29 Match the statements/expressions given in **Column I** with the values given in **Column II**.

	Column I		**Column II**
(A)	Root(s) of the equation $2\sin^2\theta + \sin^2 2\theta = 2$	(p)	$\frac{\pi}{6}$
(B)	Points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right] \cos\left[\frac{3x}{\pi}\right]$, where [y] denotes the largest integer less than or equal to y	(q)	$\frac{\pi}{4}$
(C)	Volume of the parallelopiped with its edges represented by the vectors $\hat{i}+\hat{j}$, $\hat{i}+2\hat{j}$ and $\hat{i}+\hat{j}+\pi\hat{k}$	(r)	$\frac{\pi}{3}$
(D)	Angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\,\vec{c}=\vec{0}$	(s)	$\frac{\pi}{2}$
		(t)	π

[**Ans.** (A) q, s ; (B) p, r, s, t ; (C) t ; (D) r]

[Sol.

(A) $2\sin^2\theta + \sin^2 2\theta = 2$

$\Rightarrow \quad 4\sin^2\theta\cos^2\theta = 2\cos^2\theta \quad \Rightarrow \quad 2\cos^2\theta\,(2\sin^2\theta - 1) = 0$

$\Rightarrow \quad \cos\theta = 0$ or $\sin\theta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \quad \theta = \frac{\pi}{2}$ or $\frac{\pi}{4} \Rightarrow$ **q, s**

(B) at $x = \frac{\pi}{6}$

$$\text{LHL} = \lim_{x \to \frac{\pi}{6}^-} \left[\frac{6x}{\pi}\right] \cos\left[\frac{3x}{\pi}\right] = 5\cos\frac{1}{2}$$

$$\text{RHL} = \lim_{x \to \frac{\pi}{6}^+} \left[\frac{6x}{\pi}\right] \cos\left[\frac{3x}{\pi}\right] = 5\cos\frac{1}{2}$$

$\therefore$ f (x) is discontinuous at $x = \frac{\pi}{6}$

at $x = \frac{\pi}{4}$; f (x) is constant

$\therefore$ f (x) is discontinuous at $x = \frac{\pi}{3}, \frac{\pi}{2}, \pi \Rightarrow$ **p, r, s, t**

(C) volume $= 1 \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix} 1 = \pi \Rightarrow$ **t**

(D) $\vec{a} + \vec{b} = -\sqrt{3}\,\vec{c} \Rightarrow \left|\vec{a} + \vec{b}\right|^2 = 3|c|^2 \Rightarrow 1 + 1 + 2\vec{a}\cdot\vec{b} = 3 \Rightarrow \vec{a}\cdot\vec{b} = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3} \Rightarrow$ **r**]

Q.30 Match the statements/expressions given in **Column I** with the values given in **Column II**.

	Column I		**Column II**
(A)	The number of solutions of the equation $xe^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$	(p)	1
(B)	Value(s) of k for which the planes kx + 4y + z = 0, 4x + ky + 2z = 0 and 2x + 2y + z = 0 intersect in a straight line	(q)	2
(C)	Value(s) of k for which |x – 1| + |x – 2| + |x + 1| + |x + 2| = 4k has integer solution(s)	(r)	3
(D)	If $y' = y + 1$ and $y(0) = 1$, then value(s) of y (ln 2)	(s)	4
		(t)	5

[**Ans.** (A) p; (B) q, s ; (C) q, r, s, t ; (D) r]

[Sol.

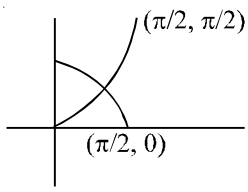
(A) $x\,e^{\sin x} = \cos x$

always increasing in $(0, \pi/2)$

x is increasing in $(0, \pi/2)$

$e^{\sin x}$ is increasing in $(0, \pi/2)$

decreasing to zero $\Rightarrow$ **p**



(B) $\begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$

$k(k-4) - 4(4-4) + 1(8-2k) = 0 \quad \Rightarrow \quad k^2 - 4k - 2k + 8 = 0 \quad \Rightarrow \quad k^2 - 6k + 8 = 0$

$k = 2, 4 \quad \Rightarrow$ **q, s**

(C) Let $f(x) = |x-1| + |x-2| + |x+1| + |x+2|$

range of $f(x)$ is $[6, \infty)$

for $f(x) = 8$, $k = 2$, $x = \pm 2$

$f(x) = 12 \quad k = 3;\ x = \pm 3$

$f(x) = 16 \quad k = 4;\ x = \pm 4$

$f(x) = 20 \quad k = 5;\ x = \pm 5 \Rightarrow$ **q, r, s, t**

(D) $\frac{dy}{dx} = (y+1)$; $\int \frac{dy}{dx} = \int dx$

$\ln(y+1) = x + C$

$y + 1 = ke^x \quad \Rightarrow \quad y = ke^x - 1$

$y(0) = k - 1 = 1 \quad \Rightarrow \quad k = 2$

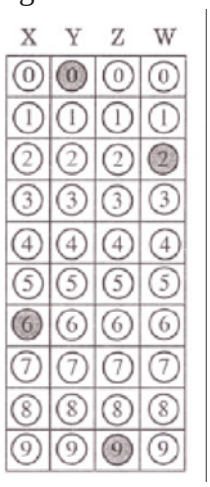
$y = 2e^x - 1$

$y(\ln 2) = 2(2) - 1 = 3 \quad \Rightarrow$ **r**]

SECTION - IV

Integer Answer Type

This section contains 8 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened. For example, if the correct answers to question numbers X, Y, Z and W (say) are 6, 0, 9 and 2, respectively, then the correct darkening of bubbles will look like the following:



Q.31 If the function $f(x) = x^3 + e^{\frac{x}{2}}$ and $g(x) = f^{-1}(x)$, then the value of $g'(1)$ is [Ans. 2]

[**Sol.** $f(x) = x^3 + e^{x/2}$ at $x = 0$, $f(0) = 1$

$g(f(x)) = x \quad \Rightarrow \quad g'(f(x)) \cdot f'(x) = 1 \quad \Rightarrow \quad g'(1) \cdot f'(0) = 1$

here $f'(x) = 3x^2 + \frac{1}{2}e^{x/2} \quad \Rightarrow \quad f'(0) = \frac{1}{2}$

$\therefore \quad g'(1) = 2$ **Ans.**]

Q.32 Let p(x) be a polynomial of degree 4 having extremum at x = 1, 2 and $\lim_{x\to 0}\left(1+\frac{p(x)}{x^2}\right)=2$. Then the value of p(2) is [Ans. 0]

[**Sol.** $\lim_{x\to 0}\left(1+\frac{p(x)}{x^2}\right)=2$

so in p(x), x^2 is a factor

$p'(x) = Ax(x-1)(x-2)$

$p(x) = A\left(\frac{x^4}{4} - x^3 + x^2\right)$

$\lim_{x\to 0}\left(1+\frac{p(x)}{x^2}\right) = 2 \quad \Rightarrow \quad A = 1$

so $p(x) = \frac{1}{4}(x^4 - 4x^3 + 4x^2) \quad \Rightarrow \quad p(2) = 0$ **Ans.**]

Q.33 Let (x, y, z) be points with integer coordinates satisfying the system of homogeneous equations :

$$3x - y - z = 0$$
$$-3x + z = 0$$
$$-3x + 2y + z = 0$$

Then the number of such points for which $x^2 + y^2 + z^2 \le 100$ is [Ans. 7]

[**Sol.** $3x - y - z = 0$ ....(1)

$-3x + z = 0$ ....(2)

$-3x + 2y + z = 0$ ....(3)

from equation (1) and equation (2)

$y = 0$ and $z = 3x$

both of which satisfy equation (3)

Let $x = k$ then $z = 3k$

$\therefore \quad x^2 + y^2 + z^2 \le 100 \quad \Rightarrow \quad k^2 + 0 + 9k^2 \le 100$

$\therefore \quad k^2 \le 10$

$\therefore \quad k \in \left[-\sqrt{10}, \sqrt{10}\right]$

$\therefore$ integer values of $k = \pm 3, \pm 2, \pm 1$ and 0 $\Rightarrow$ 7 points **Ans.**]

Q.34 Let $f : R \to R$ be a continuous function which satisfies

$$f(x) = \int_0^x f(t)dt.$$

Then the value of f (ln 5) is [Ans. 0]

[**Sol.** $f(x) = \int_0^{ln 5} f(t)\,dt$

$f'(x) = f(x)$

$\frac{f'(x)}{f(x)} = 1$

$ln\, f(x) = x + C$
$f(x) = e^x e^C$
$0 = f(0) = e^0 + C$
$f(x) = e^{x+C}$; $0 = e^C$; $C = -\infty$
$f(x) = e^x - 1$
$f(ln\, 5) = 0$ **Ans.**]

Q.35 The smallest value of k, for which both the roots of the equation

$$x^2 - 8kx + 16(k^2 - k + 1) = 0$$

are real, distinct and have values at least 4, is [Ans. 2]

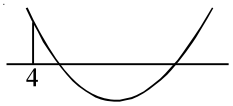
[**Sol.** α and $\beta \geq 4$

$x^2 - 8kx + 16(k^2 - k + 1) = 0$

(i) $D > 0$

$64k^2 - 64(k^2 - k + 1) \geq 0$

$k - 1 \geq 0 \quad \Rightarrow \quad k > 1$



(ii) $\frac{8k}{2} > 4 \quad \Rightarrow \quad k > 1$

(iii) $f(4) \geq 0$

$16 - 32k + 16(k^2 - k + 1) \geq 0$

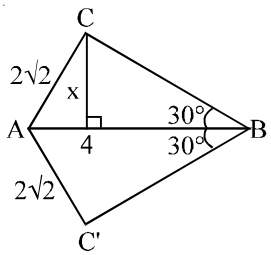
$k^2 - 3k + 2 \geq 0 \quad \Rightarrow \quad k \leq 1$ or $k \geq 2$

$\therefore$ $k \in [2, \infty)$

$\therefore$ least value of k = 2 **Ans.**]

Q.36 Let ABC and ABC′ be two non-congruent triangles with sides AB = 4, AC = AC′ = $2\sqrt{2}$ and angle B = 30°. The absolute value of the difference between the areas of these triangles is [Ans. 4]

[**Sol.** Apply cosine rule



$$\frac{\sqrt{3}}{2} = \cos 30° = \frac{16 + x^2 - 8}{2 \times 4 \times x}$$

$\Rightarrow \quad x^2 + 8 - 4\sqrt{3}x = 0$

$$x = \frac{4\sqrt{3} \pm \sqrt{48 - 32}}{2}$$

$$|\Delta ABC - \Delta ABC'| = \frac{1}{2} \times 4 \sin 30° \times BC - \frac{1}{2} \times 4 \sin 30° \times BC'$$

$= BC - BC' = 4$ **Ans.**]

Q.37 The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x \mid x^2 + 20 \leq 9x\}$ is [Ans. 7]

[**Sol.** A : $x^2 + 20 \leq 9x \quad \Rightarrow \quad x^2 - 9x + 20 \leq 0 \quad \Rightarrow \quad 4 \leq x \leq 5$

$\because$ $f(x) = 2x^3 - 15x^2 + 36x - 48$

$f'(x) = 6x^2 - 30x + 36$

$= 6(x - 2)(x - 3) > 0 \quad \forall\ x \in [4, 5]$

$\therefore$ maximum value $= f(5) = 2 \times (5)^3 - 15(5)^2 + 36 \times 5 - 48$

$= 250 - 375 + 180 - 48$

$= 430 - 423 = 7$ **Ans.**]

Q.38 The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is [Ans. 8]

[**Sol.** Let $C_1 : x^2 + y^2 = 1$

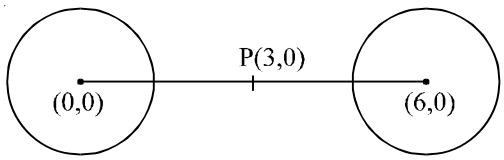
and $C_2 : (x - 6)^2 + y^2 = 1$

midpoint P = (3, 0)

let C be $(x - h)^2 + (y - k)^2 = r^2$

tangent through (3, 0) on $x^2 + y^2 = 1$, is

$y - 0 = m(x - 3) \quad \Rightarrow \quad y = m(x - 3)$



perpendicular distance from centre = radius

$$\Rightarrow \quad \frac{3m}{\sqrt{1+m^2}} = 1 \quad \Rightarrow \quad 9m^2 = 1 + m^2$$

$$\Rightarrow \quad m^2 = \frac{1}{8} \quad \Rightarrow \quad m = \pm \frac{1}{2\sqrt{2}}$$

$$\therefore \quad \text{equation is } y = \pm \frac{1}{2\sqrt{2}}(x - 3)$$

it should be tangent to C_2

$$\therefore \quad \text{perpendicular distance from } (6, 0) = \frac{\left|\pm \frac{3}{2\sqrt{2}}\right|}{\sqrt{1+\frac{1}{8}}} = 1 = \text{radius}$$

it should be tangent to C also.

$\therefore$ perpendicular distance from centre = radius

$$= \left| \frac{\pm \frac{1}{2\sqrt{2}}(h-3) - k}{\frac{3}{2\sqrt{2}}} \right| = \left| \frac{\pm(h-3) - 2\sqrt{2}\,k}{3} \right| = \left| \frac{2\sqrt{2}\,k}{3} \right|$$

$\because$ C touches C_1 and C_2 externally

$\therefore \quad CC_1 = r + r_1 \quad \Rightarrow \quad h^2 + k^2 = (r + r_1)^2 \quad ....(1)$

and $\quad CC_2 = r + r_2 \quad \Rightarrow \quad (h - 6)^2 + k^2 = (r + r_1)^2 \quad ....(2)$

Subtract (2) from (1), we get

$$12h - 36 = r_1^2 - r_2^2 + 2r(r_1 - r_2) = 0 \quad \Rightarrow \quad h = 3$$

from (1) $\quad 9 + k^2 = (r + 1)^2 \quad \Rightarrow \quad 9 + k^2 = \left(\frac{2\sqrt{2}k}{3} + 1\right)^2 = \frac{8}{9}k^2 + 1 + \frac{4\sqrt{2}}{3}k$

$$\Rightarrow \quad \frac{k^2}{9} - \frac{4\sqrt{2}\,k}{3} + 8 = 0 \quad \Rightarrow \quad k^2 - 12\sqrt{2}\,k + 72 = 0$$

$$\Rightarrow \quad k = \frac{12\sqrt{2} \pm \sqrt{288 - 288}}{2} = 6\sqrt{2}$$

$$\therefore \quad \text{radius} = \left| \frac{2\sqrt{2} \cdot 6\sqrt{2}}{3} \right| = 8$$ **Ans.**]